QUESTIONS AND ANSWERS
AND SOME BRIFE NOTES
ON
THE TEACHING OF PRIMARY
SCHOOL MATHEMATICS
(PFC 212

Q1. What is mathematics?

Ans: Mathematics like other subjects is difficult to define. It is pregnant with many definitions. Some mathematics considers mathematics as a science of number and space. Others think it is a way of finding answers to a problem. Many also think, it is the ability to think critically in order to establish relationship and use them.

Q2. Give THREE reasons why it is important to learn mathematics?

Ans: (a) To help the children to develop problem solving strategies aimed at dealing with specific problems.

(b) Equips children in finding solution to problem in everyday life and other situations such as the teaching profession, industry, law and economics.

(c) To be able to count and make simple calculation with numbers.

(d) To be able to read and understand graphs in their various forms.

(e) To know about money and be able to make simple calculations involving money.

(f) To be able to measure and do simple computations involving measurement.

(g) To be able to recognize shapes and know their properties.

Q3. Distinguish between a problem and a question.

Ans: Every problem is a question to be answered but not all questions are problems. For a question to be a problem, it must be challenging or perplexing. For example

15-7 may be a problem to a basic 2 child but not a basic 5 child. This means that what may be a problem to one child may not necessarily be a problem to another child. A problem therefore is normally considered to be a challenging or perplexing question or situation.

Q4. Distinguish between problem solving and mathematical investigation.

Answer: Problem solving is a process of finding solution to a problem or puzzling, or perplexing situation which involves the use of a known strategy to arrive at a solution. On the other hand, mathematical investigation is the process of trying things out, identifying strategies, using different approaches and methods for solving a problem one has not met before and for which there is no readily available means of solution. In other words, there is NO KNOWN strategy to arrive at solution to the problem.

Q5. State <u>FOUR</u> possible benefits pupils derive in carrying out mathematical investigation.

Answer: Mathematical investigation promotes logical reasoning in pupils. Pupils learn a lot through practical activities.

♣ Pupils understanding of mathematical processes are broadened and are able to think mathematically.

♣ Pupils interest is sustained and the ideas that mathematics is difficult and abstract are removed.

♣ Interest derived through mathematical investigations enable pupils to carry on more investigations thereby enabling them to increase their learning abilities.

♣ Mathematical investigations allow for more adaptability, flexibility and has positive transfer of learning in pupils.

Q6. State George Polya's model for problem solving in their correct order

Answer: ♥ Understanding the problem

♥ Devising a plan

♥ Carrying out the plan

♥ Looking back.

Q7. The second step in George Polya's model for problem solving is 'Devising a plan' State what a child should do before devising a plan.

Answer: ● The child should consider a wide range of approaches and make appropriate selection.

● Plan all strategies and consider what might go wrong.

Q8. The third step in George Polya's model for problem solving is "carrying out the plan" What should a child consider before carrying out the plan?

Ans: ● The child must be aware of the implications and make appropriate decisions in the light of the results obtained at various stages.

● Making generalizations and justifying them.

● Simplifying test and attempting to prove results where necessary.

● Completing the giving task.

Q9. The fourth step in George Polya's model for problem solving is "looking back" Explain what this involves.

Answer: ♥ Checking your result and finding out whether it is reasonable.

♥ Amend the result in the light of your checks on its accuracy.

Q10. (a) State George Polya's four steps for problem solving.

(b) Using a suitable example, explain each step in (a).

Answer: ● Understanding the problem.

● Devising a plan.

● Carrying out the plan.

● Looking back.

(b) given that there are 15 people and that each person should shake hands once with all other person in the group, how many handshakes shall we have altogether?

Step 1: Understanding the problem:

►This involves reading the question about three times as a means of trying to understand the demands of the question.

► This question involves shaking hands with all other people.

►Everyone will have a turn once only to shake hands with one person.

►The number of total handshake will be far more than 15.

►A person cannot shake his/her own hand. It is just impossible.

Step2: Devising a plan:

♥ You arrange the people in a straight line.

♥ Begin with a smaller number and then move to a large number. For example. 3 people will give 2 handshakes. If one person shakes hands with the remaining 2 persons and the remaining 2 also have 1 handshake each. This means for 3 people, there are 3 handshakes that is 2 + 1 = 3.

Step 3: Carrying out the plan.

♣ 2 people give 1 handshake.

♣ 3 people give 2 + 1 = 3 handshakes.

♣ 4 people give 1 + 2 + 3 = 6 handshakes.

♣ 5 people give 1 + 2 + 3 + 4 = 10 handshakes.

♣ Children can now generalize that for 15 people, the number of handshakes will be 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1= 105 handshakes.

Step 4: Looking back:

Checking to see whether the answer is correct and or can be applied in a new situation.

Is the answer reasonable or is the answer correct? Check:

Let solve back.

(1):S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14

(2):S = 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

(1) + (2) gives 2S = 15 × 14 (Note that when we reverse 14 + 13 + 12 + 11 + 10…………to 1 + 2 + 3 + 4 ………respectively and adding both figures vertically gives 15.e.g. 1 + 14 = 2 + 13 = 3 + 12 etc.) thus, $\frac{2S}{2} = \frac{15 \times 14}{2}$ = 15 × 7 = 105. Therefore the number of handshakes is 105.

Thus for n number of people, the number of handshake is (n - 1) + (n - 2) + (n - 3) + (n - 4) + (n - 5) + (n - 6) + ……… can be used to determine the total number of handshakes if each person should shake his/her friend once.

Q11. Give and explain John Mason's model for problem solving processes.

Ans: (a) Entry phase: At this phase, the problem solver asks himself several questions to determine what he knows about the question. Example, what do I know about this problem. What do I want from this problem. This is followed by the attack phase.

(b) Attack phase: At this stage, the problem solver may get stuck and through deep thinking see relationships and step on clues and exclaim – AHA!, I see what to do now. These are some of the likely feeling of the problem solver.

(c) Review phase: At this stage the problem solver checks his solution, reflects on the key ideas and the key moments and sees if the solution can be extended to a wider context.

Q12. Briefly describe a game which you could use to teach a mathematical concept of your choice. Name the basic mathematics ideas involved in the game.

Ans: A game which can be used to teach the concept of probability is ludo. It is a game played by two, three or four people. A fair die numbered one to six (1 - 6) on its faces is tossed. It is fair since each face has an equal chance of appearing when tossed. Each player uses four buttons which are of different colours usually blue, red, green and yellow. Each player tosses the die in turn and moves the buttons depending on the number that appears on each toss. A button is brought out of home when the number 6 appears after each toss. A player is first to win the game when he/she is able to remove all his buttons from home and moves round through the paths back home first.

Basic mathematical ideas involved in this game:

i) Counting

ii) Idea of possible numbers to appear (sample space)

iii) The number which appears out of the lot (event)

iv) The idea of wanting a particular number to appear out of the lot (probability)

Q13. Give <u>THREE</u> general objective of the mathematics syllabus.

Ans: ♣ Socialize and co-operate with other pupils effectively.

♣ Adjust and handle number words.

♣ Perform number operation.

♣ Collect, process and interpret data.

♣ Make use of appropriate strategies of calculations.

♣ Recognize and use functions, formulae, equation and inequalities.

♣ Recognize and use patterns, relationship and sequence and make generalization.

♣ Identify or recognize the arbitrary/standard units of measure.

♣ Use the arbitrary units to measure various quantities.

Q14. Give THREE general aims of mathematics syllabus for primary school in Ghana.

Ans: i) Develop basic idea of quantity and space.

ii) Develop the skill of selecting and applying criteria for classification and generalization.

iii) Communicate effectively using mathematical terms, symbols and explanation through logical reasoning.

iv) Use mathematics in daily life by recognizing and applying appropriate mathematical problem solving strategies.

v) Understanding the processes of measurement and acquire skills in using appropriate measuring instrument.

vi) Develop the ability and willingness to perform investigations using various mathematical ideas and operations.

vii) Work co-operatively with other pupils to carry out activities and project in mathematics.

viii) See the study of mathematics as a means for developing human values and personal qualities such as diligence, perseverance, confidence and tolerance, and as a preparation for profession and careers in science, technology, commerce, industry and variety of work areas.

ix) Develop interest in studying mathematics to a higher level.

Q15. Write down THREE points indicating the rationale for teaching mathematics in primary school in Ghana.

Ans: ► To enable pupils to reason logically.

► To function effectively in society.

► To help pupils develop interest in mathematics as an essential tool for science and research development.

► To enable pupils read and use numbers competently.

► To help pupils solve problem.

► To acquire knowledge and skills that will help pupils to develop the foundation of numeracy.

► To enable pupils communicate mathematics ideas effectively.

Q16. List SIX major areas of content covered in the primary school mathematics syllabus in Ghana.

Ans: 1. Number

2. Shape and Space

3. Measurement

4. Collecting and handing data

5. Problem solving

6. Investigation with numbers

Q17.   Distinguish between syllabus and curriculum.

Ans: Syllabus is a list of mathematics topics to be studied.

Curriculum on the other hand is the whole educational experiences of the pupils inside and outside the classroom and the school environment. It includes what is taught and how it is taught. Thus, curriculum directs the teacher on what to teach, teaching and learning materials to select and how to use them.

Q18. What are the important pieces of information the syllabus contain?

Ans: ♦ General objectives

♦ Year and units

♦ Specific objectives

♦ Content

♦ Teaching and learning activities

♦ Evaluation

♦ Profile dimension

Q19. List THREE curriculum materials that are used in primary schools in Ghana.

Ans: 1) Syllabus

2) Teacher's handbook or manual

3) Pupil's textbook

4) Other books or sources of reference relevant to the subject or topic to be treated by the teachers.

Q20. What do you understand by the term profile dimension?

Ans: Profile Dimension is a concept in the syllabus which describes the underlying behavioral change in learning during teaching, learning and assessment.

N.B: The dimensions considered are knowledge, understanding and application.

Knowledge and understanding together constitute lower profile dimension whilst application constitute higher profile dimension. The specified profile dimensions in Ghana are:

Knowledge and understanding (pry 1 – 3) 40% pry 4 – 6 (30%)

Applications of knowledge pry (1 - 3) 60% (pry 4 – 5) 70%.

Q21. Briefly explain the following terms and give example: (a) Gender (b) Gender bias (c) Gender balance.

Ans: Gender: gender refers to the different characteristics attributed to boys and girls or woman and man. It also means the different roles which society thinks boys or girls, males or females, men or woman should be assigned.

Example: Girls tend to have low interest in science and mathematics whilst men have high interest in mathematics and science as well as subject which demands higher thinking.

Gender bias: This means treating pupils unfairly based on their social roles. Example, where boys are made to study on returning from school, girls are instructed to prepare food in the kitchen or go to the market and sell. Whilst boys are given scientific toys to play with to help develop their problem solving skills, girls are given dull toys to play with.

Gender balance: (Gender equity or Gender equality) refers to a situation where pupils are given equal access to participate in any activity. Example: in the classroom situation, both boys and girls should be given equal opportunities to participate in class discussions.

Q22. Give FOUR reasons that account for less participation of girls in the study of mathematics.

Ans: 1) The perception of mathematics as a male dominated activity.

2) Boys are encouraged to be more independent than girls and this also encourage experiment and problem solving among boys.

3) Peer group pressure causes girls to fear that high attainment in mathematics will hurt the development of their relationships with boys.

4) Activities in most textbook reflect activities associated with males more than those associated with female. The same situation repeats itself in examination questions set for pupils and student.

5) In child care practice, boys are given significantly more scientific and constructional toys which encourage the development of every important mathematical concept and problem solving.

6) Schools, employers and industry appear to show little interest in encouraging or attracting girls with ability in mathematics into some of the fields and professions where there are good applicants. Boys are usually counseled to work towards a higher qualification in mathematics as a career but this is not always the case with girls.

7) In classroom, teachers tend to interact more with boys than with girls. They also give more serious consideration to boys than girls and tend to give opportunities to boys to answer more difficult higher level questions.

8) In school, more men than woman teach mathematics.

Q23. Write down THREE ways by which you would encourage girls in your class to develop interest in mathematics.

♠ Encourage group work and make girls leaders of their group, thus giving them the opportunity to talk

♠ Encourage girls to develop confidence in their mathematical abilities by motivating them through praise or complementing their good performances.

♠ Teaching learning materials used should not put girls at a disadvantage. They should be girl friendly.

♠ Plan appropriate career guidance for girls, so that they will be aware that lack of an appropriate mathematical qualification can bar them from entry into many fields of employment.

♠ Discourage boys from dominating or hijacking group and class discussions, but provides equal opportunities for both boys and girls in the discussion.

Q24. Describe briefly the Behaviorist's theory of learning.

Ans: behaviorist view all form of learning in terms of development of connections between stimuli received and responses displayed by organism or learners. Behaviorist concludes that learning is a process by which stimuli and responses bonds are established when a successful response immediately and frequently follows a stimulus. Some of the early behaviorists include Skinner, Pavlov and Thondike.

B.F Skinner's theory relates to stimulus response relationship and reinforcement. In his view learning is a change in behavior. Thus, as learners learn, their responses in terms of changed behavior increases.

Q25. Describe briefly the developmentalist's theory of learning.

Ans: Developmentalist holds the view that to understand learning; we must not confine our self to observable behavior but must be concerned with the learner's ability to mentally recognize his or her inner world of concept and memories. They believe that in learning, the mental processes of the child must be taken into account. This means children cannot learn the same content as adult and individual children will even differ in the way they learn. In other words, learning is a personal experience and the job of the teacher is to facilitate this process.

They believe that there are developmental stages in the ability of the child to think logically and that children have to reach a certain point of development before they are ready or able to understand a particular mathematical concept or the other.

Piaget, a developmentalist proposed 4 stages. These are Sensory motor stage, Pre-operational stage, concrete operational stage and formal operational stage.

Q26. Give <u>THREE</u> characteristics of pupil in the Sensory motor stage.

Ans 1) Children at this stage are mostly without language.

2)They learn to physically control objects in their environment.

3)At the latter part of this stage, children become aware of others and language begins to develop.

Q27. Give <u>THREE</u> characteristics of a child at the pre-operational stage.

Ans: 1) Children cannot carry out mentally simple operations such as addition, subtraction, multiplication and division because they are not able to relate to abstract stage of things.

2) They have difficulty with height, quantity, time and understanding space and idea of cardinal numbers.

3) They cannot classify things i.e. sort object into groups.

4) They have no reversible thought process and so cannot think logically.

5) They are good at imitating though without understanding.

6) They are self-centered and easily influenced by visual data.

Q28. Give <u>THREE</u> characteristics of children at the concrete operational stage.

Ans: a) Children begin to develop logical system of thought.

b) They achieve understanding of attribute one at a time.

c) They cannot combine a series of operations to perform complex task.

d) They begin to group things into classes and form one to one correspondence

e) They begin to understand time, quantity, space and become capable of operating independently.

f) They can reverse thought processes and order elements or events in time.

g) Cannot formulate all of the possible alternatives when they are given problem.

h) They are able to form mental actions rather than relying on the physical need to perform them.

i) They are also more comfortable with concrete materials during teaching and learning.

Q29. Give <u>THREE</u> characteristics of a child in the formal operational stage.

Ans: a) They can reverse thought processes.

b) They understand time, space, height, distance etc.

c) They can perform mathematical operations based on abstract thought.

d) They have the ability to deal with the abstract in both verbal and mathematical situations.

Q30. Describe briefly Bruner's theory of development.

Ans: Bruner's theory of development suggests that children and adults go through three main stages of development when they are learning some piece of mathematics. He states that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. He proposed three levels. Enactive, laconic and symbolic stages. In simple terms it implies that whenever a child learns new concept, he/she will pass through the concept at each of these levels in turn.

Q31. What is the implication of Bruner's theory of development for teaching a new mathematical concept?

Ans: The implication is that, the presentation of new concept should be made first using concrete materials and followed later with a gradual introduction of abstract symbols after having used picture or diagrams (learning proceeds from concrete – semi concrete - abstract). It is to be noted that the proportion of time spent on each of these stages will vary according to age, experience and ability i.e. younger children will require lots of concrete activities to understand new concepts which may not be the case with older children though some models may still be useful to them.

Q32. Explain briefly Jerome Bruner's Enactive stage of development in learning.

Ans: At this stage the child uses motor responses to past events. The child at this level manipulates concrete materials directly. The child learning involves direct experience and manipulative skills. Example: In performing addition sums: three plus four, a child has to form a set of four objects and another three objects and put them together before determining that there are seven objects in all.

Q33. Explain briefly Jerome Bruner's "laconic" stage of development in mathematics learning giving example.

Ans: It is the stage where the child uses mental pictures of concrete objects. At this stage the child uses mental image to represent a concept. This is based on visual data such as pictures, diagrams and illustrations. Example, a child is able to picture mentally that a collection of two pens added to five pens give seven pens.

Q34. What do you understand by the term CONCEPT?

Ans: concept refers to the knowledge gained through experience or activities. Concept is formed when the meaning of a topic understudy is fully understood so as to apply the idea gained in a new situation.

Q35. How does Richard Skemp describe concept?

Ans: According to Richard Skemp, concept is the mental object which results when we abstracts from something which they all have in common a number of times.

For instance, young children learn the concept of "dog" from repeated contact with many types of dogs. Also children learn the meaning of number by experiencing number in many varied situations or through suitable collection of examples. The same holds for addition, subtraction, multiplication, division and measurement.

Q36.. In the teaching and learning of mathematics, the child learn "concept" and "skills" Explain these two terms.

Ans: concept refers to the knowledge or mental image or idea gained through experience or activities mathematically concept is formed when the meaning of a topic under study is fully understood so that the knowledge gained can be used in a new situation.

Example the symbol "3" represent a mental image of all things or groups containing three items such as illustrated in the brackets (•••) the concept of dogs is formed from repeated contact with many dogs. On the other hand skill is the mechanical aspect of mathematics. It is the ability to do something. How to add. Subtract, divide or multiply are examples of skills.

Q37. What is meant by primary and secondary concept?

Ans: Primary concepts are the knowledge or ideas built from sensory experiences. For example, seeing, feeling, smelling, tasting and touching. The concept of "redness" for example is formed from seeing many red object and recognizing their common property. Also the concept of "threeness" or twoness is formed from experiencing many different sets each containing three or two objects respectively.

Secondary concepts are knowledge or ideas built by combining primary concepts. Example, words such as red, blue, green, etc are used to describe colours. One, two, three etc are referred to as number. In this case colour and numbers are referred to as secondary concept.

Q38. A) Explain briefly what is meant by a fraction.

B) State THREE activities that will lead to the concept of a fraction.

Ans: A) A fraction is the result of dividing something into a number of equal parts and each of the original whole is a fraction. Thus a fraction is a part of a whole.

B) The activities leading to the concept of a fraction.

- Folding a sheet of paper into equal parts and colour the parts with different colours.
- Finding how many times the content of a smaller container can be obtained in a bigger container.
- Grouping a set of objects into subsets and selecting a subset out of the bigger set.

Q39. Distinguish between a common/proper fraction and improper fraction.

Ans: A common or proper fraction is a number less than 1 or a fraction whose numerators is smaller than the denominator. Example $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ etc.

On the other hand, an improper fraction is a number greater than 1 or whose numerator is smaller than the denominator. Example $\frac{7}{5}$. $\frac{5}{4}$ $\frac{4}{3}$.

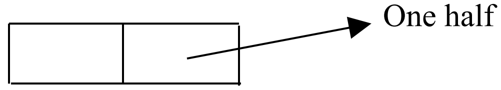
Q40. What is an equivalent fraction?

List THREE pairs of fraction that are equivalent.

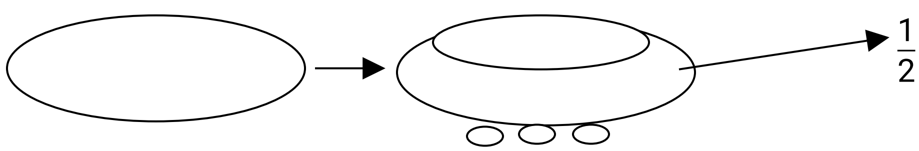
Ans: Two or more fractions are equivalent if they are different numerically but are the same in value. $\frac{2}{4}$ and $\frac{4}{8}$, $\frac{1}{3}$ and $\frac{2}{6}$.

Q41. How would you use real life situation in introducing children in basic 2 to the concept of a fraction.

Ans: A) Sharing (part – whole model): this is the case where children break or cut items like oranges or sweets into bits and share.



B) Part group model: considering part of a set in relation to the major set. E.g. group a set into a subset and selecting a subset out of the major set.



B) Ratio model: This shows the relationship between objects of the same kind. Thus comparing objects of the same kind. Fraction as a division of two numbers. E.g. $\frac{3}{8}$ as 3:8.

Q42. Explain how you would use concrete materials to show to a basic 4 pupils that the fractions $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent.

Ans: a) using a paper folding and shading:

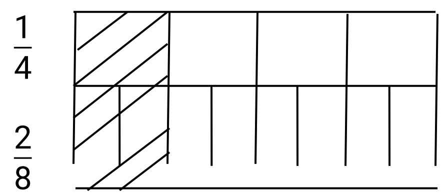
Fold a sheet of paper into four equal parts and shade one part.

Fold another sheet of paper of the same dimension into eight equal parts and shade two.

Guide pupils to compare the shaded portions of the two sheets of the papers by laying them side by side and end to end.

Pupils see that, the area of the shaded portion for $\frac{1}{4}$ and $\frac{2}{8}$ are the same.

Guide pupils to write this as $\frac{1}{4} = \frac{2}{8}$



b) Using Cuisenaire rod:

♥ Pick a brown rod as a whole.

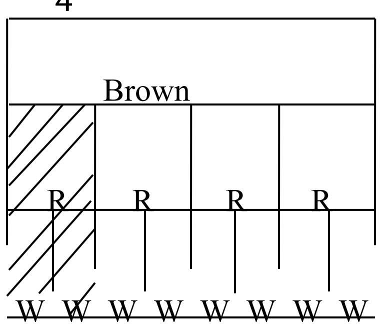
♥ Split the brown rod into 4 red rods and pick one red to present $\frac{1}{4}$

♥ Now split the brown (whole) rod into 8 white.

♥ Guide pupils to pick 2 white rods out of the 8 white.

♥ Explain that the 2 white represent 2 out of 8 white rods.

NB: $\frac{1}{4}$ represents 1 red rod out of 4 red rods. Thus, $\frac{1}{4} = \frac{2}{8}$



1R represents one red out of 4 red = $\frac{1}{4}$

1 R = 2w = 2 out of 8w = $\frac{2}{8}$

Thus, $\frac{1}{4} = \frac{2}{8}$

Q43. Explain as clearly as possible using diagrams and words how you would demonstrate to class 4 pupils which is greater fraction in each of the following examples.

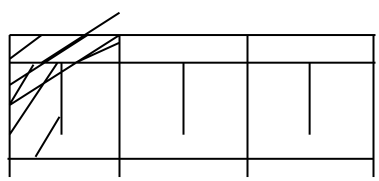
a) $\frac{1}{3}$ and $\frac{1}{6}$ b) $\frac{4}{5}$ and $\frac{3}{4}$

Ans: (a) ❖ Let pupils cut two rectangular strips of papers having the same size and area.

❖ One cut out is divided into three equal parts and one part is shaded for $\frac{1}{3}$ whilst the other part is divided into six equal parts and one cut out shaded for $\frac{1}{6}$.

❖ Pupils put the cut-out side by side and end to end to compare their areas as shown below.

$\frac{1}{3}$



$\frac{1}{6}$

❖ From the diagram, pupils see clearly that the area for $\frac{1}{3}$ is greater than the area for $\frac{1}{6}$

❖ Guide pupils to write this as $\frac{1}{3} > \frac{1}{6}$ i.e. $\frac{1}{3}$ is greater than $\frac{1}{6}$ in value.

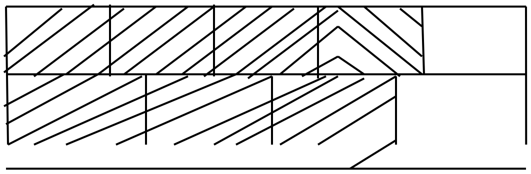
Ans: b) ❖ Two rectangular cut - outs of the same size and area are supplied to pupils.

❖ One cut out is divided into five equal parts and four parts shaded for $\frac{4}{5}$.

❖ Let pupils fold the other paper into four equal parts and three parts shaded for $\frac{3}{4}$

❖ Guide pupils to compare the area of the shaded portions by laying them side by side and end to end.

❖ Pupils discover that the area covered by $\frac{4}{5}$ is greater than $\frac{3}{4}$. Thus $\frac{4}{5} > \frac{3}{4}$.



$\frac{4}{5}$

$\frac{3}{4}$

N.B: Other TLM's such as Cuisenaire rod and fractional charts can also be used.

Q44. Use concrete materials to explain to basic five pupils the addition of fractions $\frac{1}{5} + \frac{2}{5}$

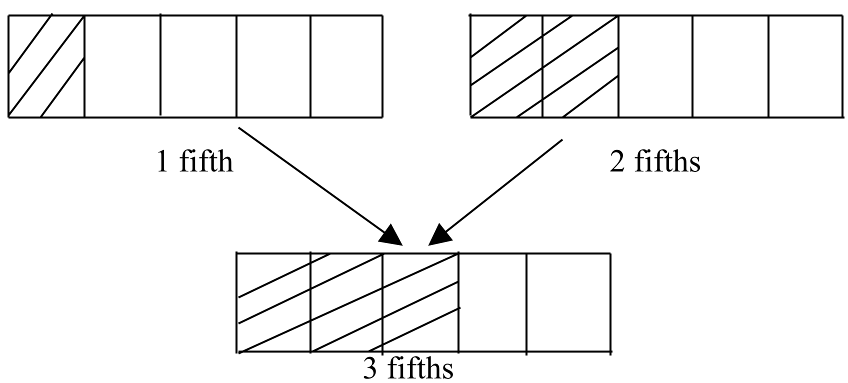
Ans: ¿ Let pupils take a rectangular sheet of paper

¿ Guide pupils to fold into five equal parts and shade one part for $\frac{1}{5}$

¿ Take another rectangular sheet of paper of the same dimension.

¿ Fold into five equal parts and shade two parts for $\frac{2}{5}$

- Let pupils put the two shaded portions together to give three portions as shown below.



- Guide children to write this as $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$

b) Using Cuisenaire rods:

² Take a yellow rod as a whole.

² Split the yellow rod into five whites.

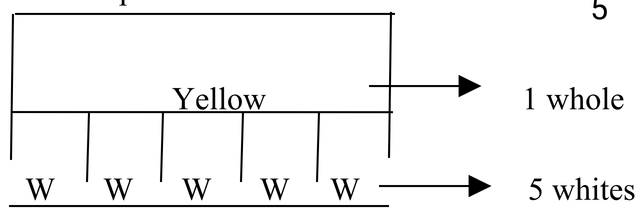
² Pick 1 white rod to represent $\frac{1}{5}$

² Pick another 2 white rods to represent $\frac{2}{5}$

² Add the 1 white rod to the other 2 white rods to give 3 white rods and compare with the five white rods.

[2] Pupils infer that the 3 white rods represent 3 out of the 5 white rods i.e. $\frac{3}{5}$

Pupils write this as $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$



3 whites compared to a whole (5 whites) = $\frac{3}{5}$ .i.e. $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

Q45. a) Explain how you would guide a class 5 pupil to add the following fraction $\frac{3}{8}$ and $\frac{2}{8}$ using a named concrete material.

b) Explain how the algorithm for the addition of fraction will be developed.

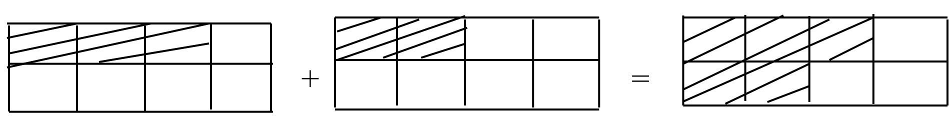
Ans: ❖ Fold a rectangular sheet of paper into 8 equal parts and shade 3 for $\frac{3}{8}$

❖ Fold another rectangular sheet of paper of the same dimension into 8 equal parts and shade 2 parts for $\frac{2}{8}$.

❖ Add the two shaded portions and compare with a whole.

❖ Pupils see that the addition of the shaded portions as $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Thus



$\frac{3}{8}$ $\frac{2}{8}$ $\frac{5}{8}$

Algorithm: $\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$

N.B. Cuisenaire rods can also be used.

Q46. Using the example $\frac{1}{3} + \frac{1}{5}$, describe step by step how you would use Cuisenaire rod to develop algorithm for the addition of common fraction with class 4 pupils (DBE 2007)

Ans:
- Guide pupils to identify the whole as 1 blue and 1 dark green put end to end.
- Guide pupils to split the whole into 3 yellow rods (i.e. into thirdths) and take 1 yellow for $\frac{1}{3}$.
- Guide pupils to split the whole into 5 light green rods and pick 1 light green rod for $\frac{1}{5}$.
- Guide pupils to split the whole into 15 whites.
- Let pupils put the light green and 1 yellow rod together.
- Since this is impossible, guide pupils to exchange 1 light green for 3 whites and 1 yellow rod for 5 whites.
- Let pupils put the 3 whites and the 5 whites together to get 8 whites as

3 whites + 5 whites = 8 whites

Thus $\frac{3 \text{ whites}}{15 \text{ whites}} + \frac{5 \text{ whites}}{15 \text{ whites}} = \frac{3 \text{ whites}+5 \text{ whites}}{15 \text{ whites}} = \frac{8 \text{ whites}}{15 \text{ whites}}$.

Algorithm: $\frac{3}{15} + \frac{5}{15} \quad \frac{3+5}{15} = \frac{8}{15}$

(N.B rectangular sheet of paper can also be used).

Q47. What is a mathematical abstraction?

Ans: To learn a concept, children require a number of common experiences relating to the concept. Thus finding the common property of several seemingly disconnecting examples are called abstraction.

Q48. State any <u>THREE</u> disadvantages of instrumental learning.

Ans:
- ❖ It does not aid discovery among children.
- ❖ It does not encourage logical thinking among pupils or children.
- ❖ No practical approach to teaching, thus pupils forget easily what they learnt.

Q49. State any THREE advantages of instrumental learning.

Ans: ² A greater percentage of the course content is covered within a short time.

² It is easy to acquire and is usually preferred by students especially when the date for writing an examination is very close. E.g. student just need the quadratic equation formular to solve problems and not to prove it.

² Less knowledge and thinking is required.

Q50. State at least THREE advantages of relational learning.

Ans: ¿ It is more adaptable to new tasks. For instance if a child has relational learning, he or she will not only know which method works but why it works as well. This enable him or her to relate the method to new problems.

¿ Mathematics learnt relationally is easy to remember. That is the understanding of interrelations between concepts allow them to be remembered as part of a connected whole instead of a list of separate rules.

¿ Relational learning can be effective as a goal in itself. The need for external reward and punishment is greatly reduced thus making the motivational side of a teacher's job much easier.

¿ Relational understanding /learning is organic in quality. When people get satisfaction from relational understanding, they do not only understand material put before them but also actively seek new material and explore new areas.

Q51. State TWO disadvantages of relational learning.

Ans: ♥ It is not cost effective since much money will be required to purchase materials for practical activities.

♥ It is time consuming and therefore not enough content can be covered within the given time by the product.

Q52. Explain what is meant by the term Multiple Embodiment principle and give examples.

Ans: The principle of Multiple Embodiment refers to situation whereby each concept is represented by the use of different models, examples, different approaches and situations, thus helping to abstract the concept of a given idea without their perceptions based on only one or two specific concrete examples. For example using paper folding and shading in introducing fractions, using Cuisenaire rods in producing fractions, finding how many times the contents of a smaller container could be obtained in a bigger container in introducing fraction. Grouping a set into subsets and picking a subset to indicate a fraction.

Q53. Explain what is meant by conservation.

Ans: conservation refers to the ability of a child to detect that a measure remains the same or intact whether it's elements are put together or separated from each other.

Q54. 1) What do we mean when we say that a child has "conserved number"?

2) Briefly describe a conservation test you would carry out to determine whether a primary class 1 pupil has conserved number or not.

Ans: The ability of a child to tell that the number of object in a group remains the same when the object in the group are re-arranged or separated.

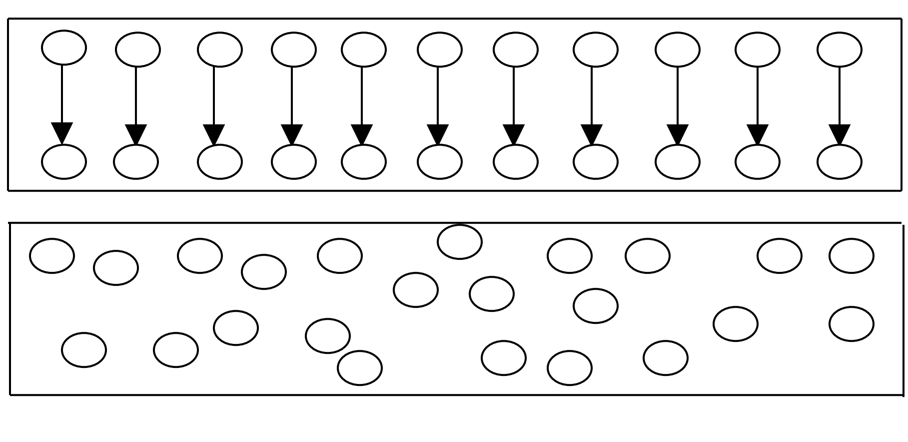
Example: 1) Teacher shows grains in his/her palm to the pupils. As the pupils observe, the teacher pours out these grains on the floor or on a table so that they spread out.

2) Teacher asks pupils whether the grains on the table or floor are more than, less than or as many as the quantity of grains which were in his palms without counting.

3) The child is said to have conserved number if he/she says that the number of grains on the floor are the same as when they were in the palm without counting.

ii) To see if a child has conserved number.

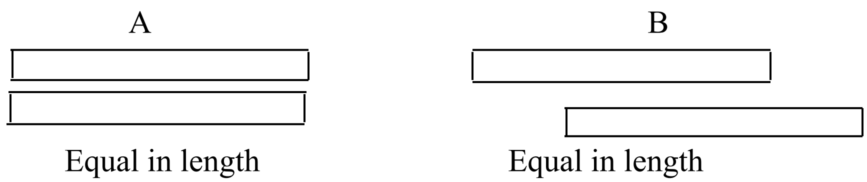
Show children two groups of counters: First line the groups up so that there is an obvious one correspondence as shown.



Next, ask the children if the two groups have the same number of objects. If the children say yes spread the objects in one of the groups apart. Ask the children if the two groups are still the same. If the child believes that one group has more objects, the child has not conserved number.

Q55. How would you detect that a child has conserved length?

Ans: when stick of equal lengths are either placed together or separated and a child says the stick are equal they have conserved length.



No increase or decrease has taken place on any of the sticks, only that one has been shifted slightly away from the other. If the child sees that group A and B all have equal length, then the child has conserved length. If the child says that they are not equal, then the child has not conserved length.

Q56. Explain what is meant by conservation of weight/mass.

Ans: Take 2 tins of the same shape and volume, using beam balance, let children observe that both weights are equal.

Disfigure one of the tins by knocking its side with a stone or hammer and ask the child whether the two tins still have the same weight/mass.

If children observe that the weight still remains the same despite the changes in shape and volume, the children have conserved mass or weight. Otherwise they have not conserved mass or weight when they say that one weighs more than the other.

Q57. A) What is sorting? OR What does a child do when she sorts objects.

B) Give <u>TWO</u> reasons why you would engage a child in K.G or class 1 in sorting activities.

Ans: A) Sorting is grouping or separating objects according to identifiable characteristics, attributes or properties and can be distinguished from all others.

Example: separating objects into colours, size, shapes etc.

B) Reasons:

i) The child can identify similarities and differences among objects.

ii) Before the child can count, he/she should be able to identify the objects to count.

iii) To provide the child with the basis for building early number concepts.

iv) As a pre-requisite for the formation of sets.

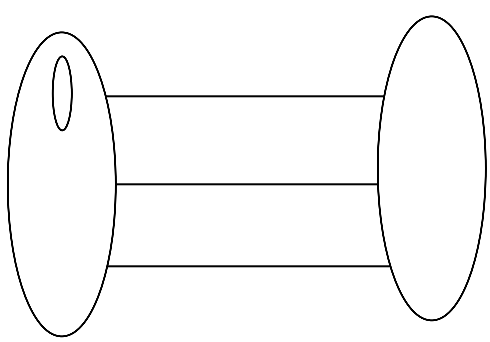
Q58. Explain the following terms as used in pre – number activities.

a) Ordering b) Matching c) Comparing

Ans: a) Ordering of a set is the arrangement of a given set into the same logical order such as order of magnitude, weight. E.g. asking pupils to make a straight line standing in order of height with the shortest pupil first up to the longest pupil. Pupils collect a number of cylindrical empty tins and arrange them in order of size. Ordering thus involves comparing objects to determine which one is shorter or longer than the other or which is heavier or less heavier than the other thus, the accompanying vocabulary or concept such as longer than, heavier than etc. are learnt.

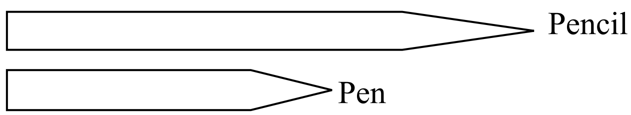
b) Matching involves pairing members of one set with members in another set in a one to one correspondence

Matching two different sets as shown below.



Match a group of girls to a group of boys so that each has a partner.

c) Comparing also means bringing two different objects on a flat base and noting the difference between them. Example comparing the length of pencil and pen.



Q59. A child in kindergarten class 2 (K. G2) Compares the length of a pencil and pen give <u>THREE</u> expressions which the child can use to describe the lengths.

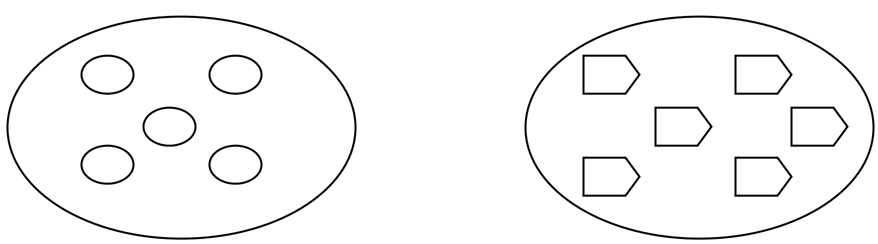
Ans: ^{2} The pencil is longer than the pen.

^{2} The pencil is shorter than the pen.

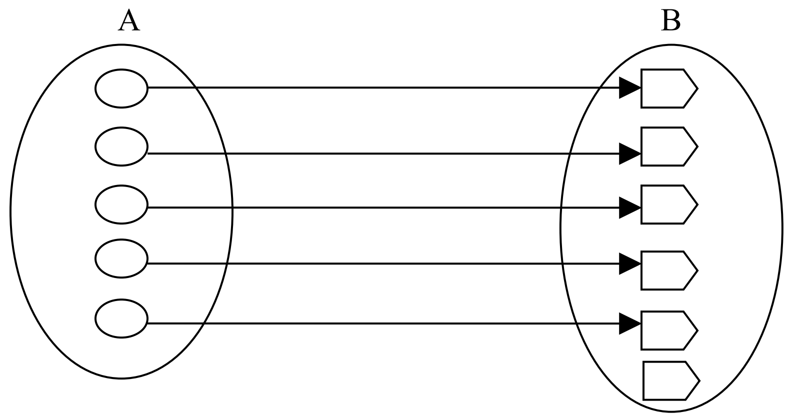
^{2} The pencil as long as the pen.

^{2} The pen is shorter than the pencil.

Q60. Describe how you would guide a child who cannot count to determine which of the two groups of object A and B below is bigger.



Ans: ● Guide pupils to match or pair the members of these two sets in one to one correspondence.



- Pupils note that a member in set B has not been matched.
- Pupils conclude that set B is bigger than set A.

Q61. Given THREE containers E.F and G, describe briefly how you would guide an early childhood pupil to order the containers according to their capacities.

Ans: ♥ Guide pupils to compare the capacities of two containers at a time.

♥ Order the containers according to their capacities.

♥ This involves comparing E and F to determine which is larger.

♥ Next compare E and G to determine which is larger.

♥ Then compare F and G to determine which is larger.

♥ Then arrange them in order from less heavier to the heaviest.

Q62. a) What is counting?

b) Why would you teach a child how to count?

c) What relevant skills should a child have to be able to count objects in a group efficiently?

d) Describe in detail the steps you would take a pupil in K.G through to enable him to count the object in a group correctly.

Ans: a) counting is using a sequence of words (one, two, three, four …….) to correspond with the items to be counted. The items could be bottle tops or beans.

b) A child is taught how to count so that the child can determine the number of objects in a given group and tell how many objects there are in a group.

c) To know the number name in the conventional order i.e. one, two, three……… and be able to match these words one by one to the objects being counted. i.e. coordinating the number names and the group of objects.

d) i) Explain that when we count the objects in a group, we determine how many object there are in the group.

ii) Guide pupils to pick an object in the given group and place it in a new position and say the number name corresponding to the number of object in the conventional order beginning with one, that is if he has one object at the new place, he says one.

iii) If there are two objects at the new place he says two (that is, there are two objects there)

iv) Guide the child to continue with this activity until all the objects at the original place have been moved to the new place.

v)The child uses the last number name mentioned to describe the number of objects in the group that is the number name tells how many objects there are in the group (cardinal number)

Q63. a) Explain the difference between a cardinal number and an ordinal number.

b)Describe <u>TWO</u> activities (one for each aspect) which you can carry out with primary class pupils so that they will understand the following aspects of number

i) Cardinal number

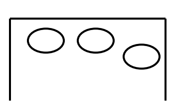
ii) Ordinal number

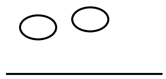
Ans: (i) A cardinal number tells how many objects there are in a set e.g. there are 40 students in class 2. 40 is used in the cardinal sense. Whilst an ordinal number gives the position of an object in relation to other objects in the group. Evans was third in the 100m race competed by 10 students.

(ii) Activities that will enable primary class 1 pupils to understand cardinal number are:

a) Put some objects on the teachers table and ask them to tell how many objects there are on the table.

b) Prepare number tray or boxes with different objects in them and ask children to indicate the number names and numerals accordingly.



 five

c) Activities that will enable primary class 1 pupils to understand the ordinal number

i) Arrange children in order of height, weight, or size and number them accordingly. From the arrangement a child realizes that his friend is either longer or shorter than him or her.

ii) Thus a child infers that he is 1^{st}, 2^{nd}, 3^{rd} or 4^{th} in the arrangement.

iii) Give children successive numbers as they enter the classroom so that children will know who entered the classroom first, second, third etc. Thus, children infer that one of them e.g. Kwasi was the first among the first four children who entered the classroom.

Q64. Explain the concept zero to a pupil in class1.

Ans: ►Children may have problem with the number zero because zero is the number property for an empty set.

♦ Guide children to understand the concept zero by asking them to identify a group of pupils with two heads in their class or a group of pupils taller than the school building.

♦ Pupils find that each of the group have no members.

♦ Inform pupils that the number for group is written as "0" and the number name is zero.

Q65. Describe briefly how you would help a kindergarten 1 (K.G) pupils to understand clearly the concept of the number "two".

Ans: ►Show group of various objects each containing two objects to the pupil.

►In each case, call out the number name "two".

►Next ask pupils to show you two objects. e.g. two pencils, two book, two eyes.

Q66. Briefly describe two activities in which you would engage a child in an early childhood to enable him write the numeral 3

Ans: ♣ Drawing the numeral 3 on sand with finger.

♣ Writing the numeral 3 in the air.

♣ Tracing the numeral 3 from stencil or cut out.

Q67. Describe an activity in which you would engage a K.G pupil in each of the following way.

a) Pattern.

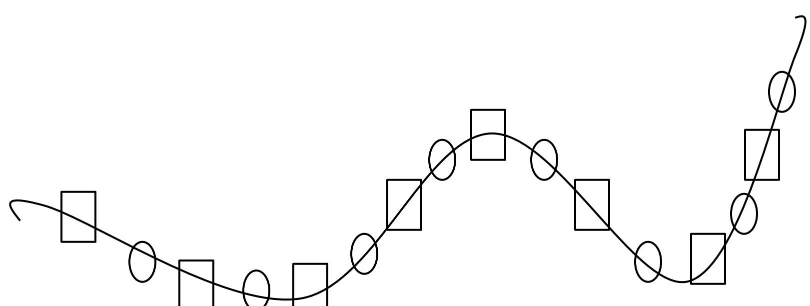
b) Finding the next term in a pattern.

Ans: a) Pattern.

►Show a pattern made of concrete material to pupils.

►Ask pupils to make one just like the one shown using concrete material

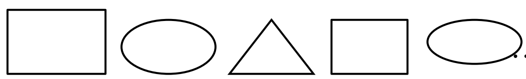
►Give pupils strings with beads and ask them to make the same pattern.



b)Finding the next pattern.

►Find the next pattern made with the given pattern.

►Pupils study the pattern made with attribute shapes to find the next shape in the pattern.

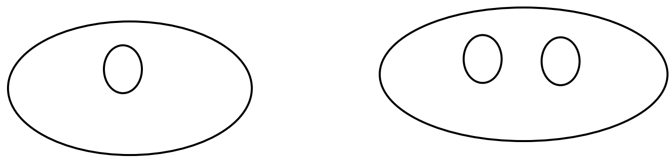

…………………..

Q68. How would you use concrete or semi – materials to class 1 pupils to perform the following operations.

a) 3 + 4 b) 37 + 48

Ans: using counters

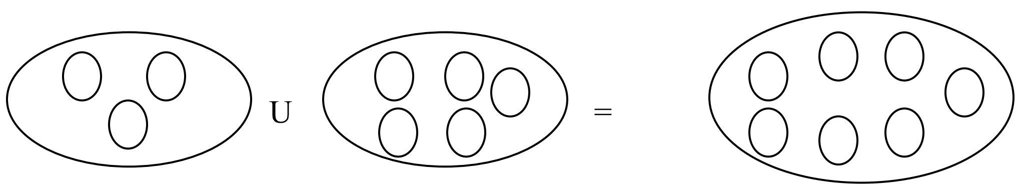
♥ Let children from a set containing 3 counters and another set containing 4 counters is shown



♥ Ask pupils to put the objects in the first set and the second set together.

♥ Ask pupils to count to determine the number of objects put together.

♥ Introduce the union U, equal to sign and the plus sign.



♥ Discuss with children that three objects together with four objects gives seven objects.

♥ Introduce the formal language and notation of addition to children

♥ Thus "three plus four equals seven" 3 + 4 = 7.

Using Cuisenaire rods:

■ Guide children to pick a light green rod which represent 3 and pick another purple rod which represent 4.

■ Guide children to place the two rod end to end.

■ Guide children to find another rod that will match the total length.

■ Children discover that black rod which represent seven matches the total length.

■ Pupils thus notice that 3 + 4 = 7 and read as three plus four equals seven

Using of number line (semi concrete materials)

■ Ask children to draw straight line on the floor.

■ Guide children to work equal spaced points on the line.

■ Let children assign whole numbers to the desired points with the point "0" starting from the left end.

■ Guide children to step on zero (0) and jump three steps forward facing right to 3 and again jump four 4 steps forward still facing right to point 7.

■ Children notice that 3 + 4 = 7.

N.B. (use several examples of additions of whole numbers using number line for pupils to realize that the operation sign plus (+) means movements)

Children need to know the basic addition facts thoroughly before they are introduced to additions of whole numbers with two or more digits.

Help children to understand the concept of place value. Place value is the value of a digit in a number depending on its place or position in the given number.

e.g. in the number 54, the "5" is in fact 50 or 5 tens while the "4" is 4 ones.

Also 67, 6 tens + 7 ones. Similarly 248 means 200 + 40 + 8

i.e. 20 tens + 4 tens + 8 ones

2 flats + 4 longs + 8 cubes

The value of 2 in the number 248 is 200, the value of 4 in the number is 40 and the value of 8 is 8 ones.

The following base ten TLM may be used to help children add whole numbers with two or more digits.

a) Dienes base ten materials.

b) The abacus.

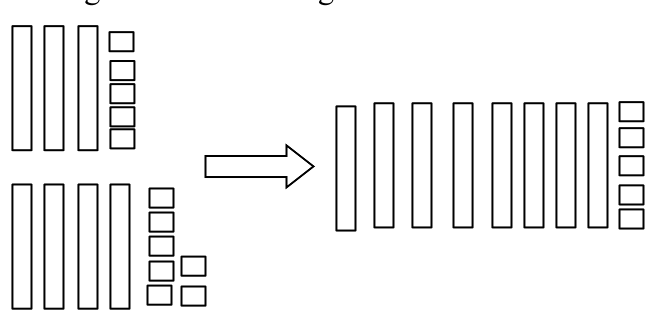
c) Bundles of sticks

a)Using Dienes base ten materials.

►Let children represent 37 by 3 longs and 7 cubes and 48 by 4 longs and 8 cubes.

►Guide children to put these together so that they get a total of 7 long and 15 cubes.

► Guide children to regroup or exchange the 15 cubes to get 1 long and 5 cubes so that in all children get a total of 8 longs and 5 cubes.



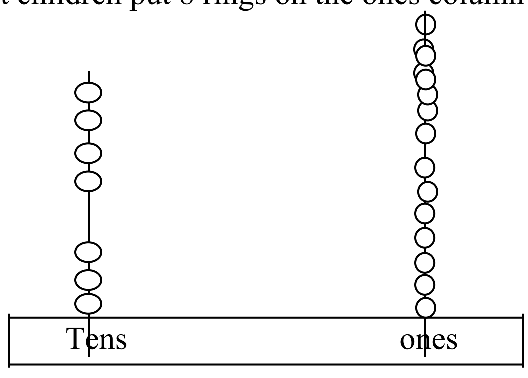
Children see that 37 + 48 = 85

b) Using Abacus:

►Explain that the rods on the right and left of the abacus are ones and tens respectively.

►With 37, let children put 7 rings on the ones column and3 rings on the tens column

►With 48, let children put 8 rings on the ones column and 4 rings on the tens column below :



►Let children identify that there are more than ten rings in the ones column.

►Explain that 10 rings on the ones column represent 1 ring on the tens column.

►Guide children to remove 10 rings from the ones column and replace them with one ring on the tens column.

►Children then realize that there will be 8 rings on the tens column and 5 rings on the ones column.

►Guide children to write this as 37 + 48 = 85.

N.B. Use similar examples and extend to addition of 3 or more digits and explain that the rods on the abacus from right to left are ones tens hunbreds thoudsands respectively.

ii) Using bundles of sticks.

♦ Guide children to represent 37 by three bundles of ten sticks and seven single sticks and the 48 by four bundles of ten sticks and 8 single sticks.

♦ Let children put them together so that they get a total of seven bundles of ten sticks and 15 single sticks.

♦ Combine the single sticks to get 8 bundles of ten sticks and 5 single sticks.

♦ Children count the bundles of sticks to get eight bundles of ten sticks and 5 single sticks.

♦ Children realize that they have 8 tens and 5 ones which they write as 37 + 48 = 85

Now try this question.

a) Using the number 243 explain what is meant by the term "place value" of the digits 243.

b) Describe how you would guide primary class 3 pupils to find the sum of 36 + 47 using Dienes base ten materials (DBE 2008).

Q69. Describe <u>THREE</u> different ways you would use to help pupils in primary class one to solve the problem 6 – 2.

Ans: There are three ways by which subtraction occurs. These are take away, comparison and missing added.

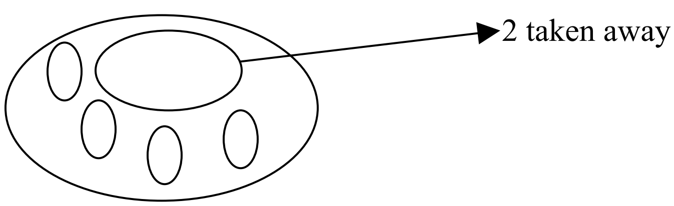
Take away means identifying a smaller group within a larger group and removing the smaller group from the larger group.

In using take away approach:

▬ Let pupils form a set containing six objects.

▬ Let pupils remove two objects from the set of six objects.

▬ Ask pupils to tell how many objects are left as the answer.



Thus, six take away two equals four.

▬ Pupils then write this as 6 – 2 = 4.

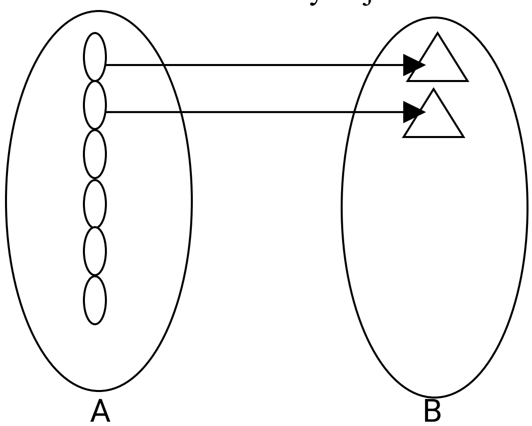
Comparison involves matching two sets in one to one correspondence and finding out how many members are left in the larger set.

In using comparison approach.

▬ Guide pupils to form a set[A] containing six objects and another set[B] containing two object.

▬ Guide them to match objects in set A to the objects in set B in a one – one corresponding.

▬ Ask them to tell how many objects are left in set A as the answer.



Thus six objects matched to two objects equals four. Children then write it as 6 – 4 = 4.

Missing addend requires finding how many more must be added to a set to make it up to a given number.

In using this approach.

▬ Guide pupils to find how many objects must be added to two to make up six.

▬ Children find that four must be added to two to make up six

Thus 6 – 2 = 4.

N.B Cuisenaire rods.

▬ Guide the child to pick a dark green rod which represents a whole with the value six.

▬ Let pupils pick another red rod which has a value of 2 and find another rod which when put to the end of the red rod will be equal to the dark green rod.

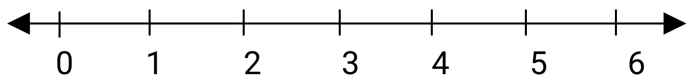
▬ Children find this to be purple. Pupils find that 4 is the missing number that must be added to 2 to be equal to six.

Number line[semi-concrete] can also be used to solve the problem 6 – 2

In using number line.

▬ Guide pupils to draw a number line on the floor.

- Let children mark equal point on the line.
- Let children assign whole numbers to the desired points with the point "0" starting from left hand as shown:



- Guide pupils to step on zero point and turning right, jump 6 steps forward and land on 6.
- A child now at 6 is guided to move backwards 2 steps to land on 4.
- Child realizes that 6 – 2 = 4

Subtraction can be extended to two or more digits whole numbers.

Q70. How would you guide a pupil in class three to solve the problem?

a) 47 – 26 b) 36 – 17

Ans: a) 47 – 26 can be solved using Dienes base ten material, abacus and bundles of sticks. The idea of place value is also required.

In using bundles of sticks:

Guide children to represent 47 as 4 bundles of ten sticks and seven single sticks.

Guide children to take away two bundles of ten sticks from the four bundles of ten sticks leaving two bundles of sticks and ask them to take away 6 single sticks from the seven single sticks.

Children realize that after the activities two bundles of ten sticks and one single stick are left.

Thus, they write this as

4tens 7 ones

- 2tens 6ones

2tens 1one.

Therefore 47 – 26 = 21

b) 36 - 17

Using Dienes base ten materials:

Guide pupils to represent 36 by 3 longs and 6 cubes

Guide children to take away 1 long from the 3 longs.

Guide children to break 1 long into 10 cubes so that in all we have 1 long and 16 cubes.

Let pupil take away 7 cubes from the 16 cubes living 9 cubes. Therefore 36 – 17 = 19.

Children are then guided to write this as 36 – 17 =19

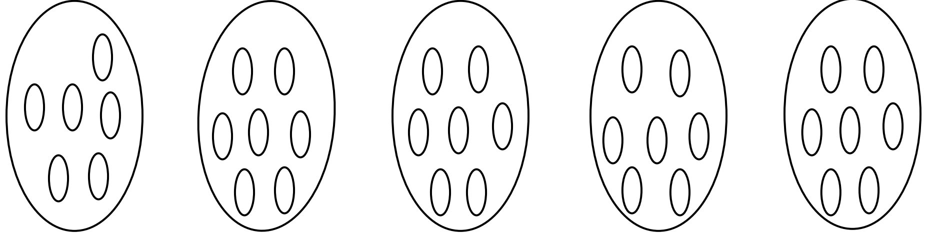
Dear learner, use abacus and bundles of sticks to solve the above problem as your assignment.

Q71. a) Draw diagram to illustrate the two multiplications 5 × 7 and 7 × 5.

b)Using the same interpretation, explain each diagram.

c)give one other way of interpreting multiplication.

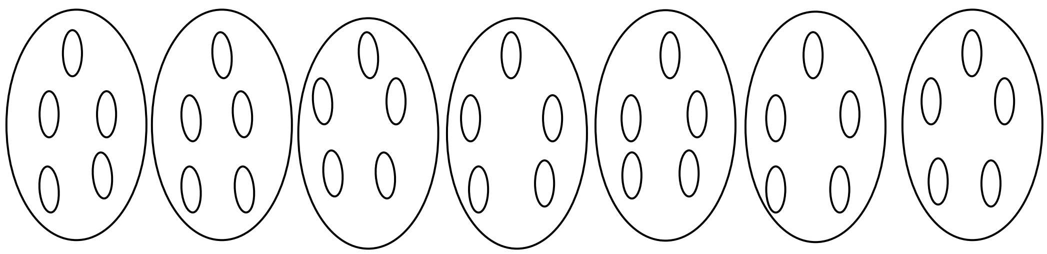
Ans: Ask pupils to form five sets of seven members each. That 5 × 7 can be thought of as 5 groups of seven.



Ask pupils again to put together all the members of the set formed and count them.

Pupils count the members together and discover that 5 lots of 7 = 7 + 7 +7 + 7 + 7 (repeated addition)

Ask pupils again to form seven sets of five members each.



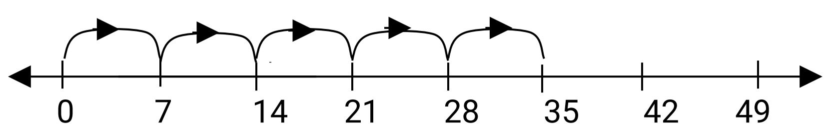
Pupils put together the sets formed and court the elements.

Pupils discover that 7 lots of 5 = 5+5+5+5+5+5+5 = 35

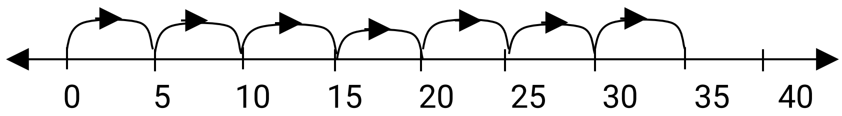
Alternative solution:

Number line can also be used.

- Let pupils draw a number line on the floor.
- Mark on the number line whole numbers beginning from zero at intervals of 7 and hop 7 units repeated 5 times to land on 35.
- Pupils see that 5×7 = 35



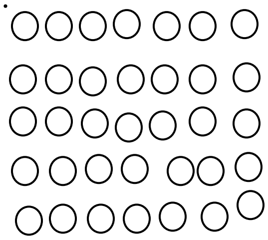
- Guide pupils again to draw another line and mark on it whole numbers starting with zero at intervals of 5.
- Guide pupils to step at zero and hop 5 units repeated seven times to land on 35.



■ Pupils see that 7 × 5 = 35.

■ Thus they conclude that 5 × 7 = 7 × 5.

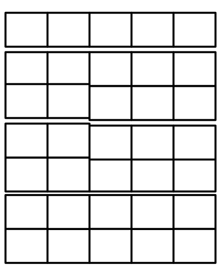
Another method of representing multiplication of whole numbers is with a rectangular array of objects.



5 × 7 = 35

Which is the same as 7 group of 5 or 7 fives.

■ Pupils arrange the objects as illustrated above to look like a rectangle.



7 × 5 = 35

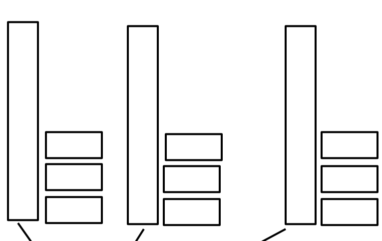
Pupils count the small boxes and get 35.

■ Thus they realize that 7 × 5 = 35.

■ They then conclude that 5 × 7 = 7 ×5.

Q72. How would you help upper primary five pupils to solve 3 × 13 using Dienes base ten materials.

Ans: ►Guide pupils to represent 13 by 1 long and 3 cubes three times.



►Guide pupils to put the long and ones together and count.

►Pupils get 3 longs and 9 cubes.

►Thus pupils observe that 3 × 13 = 39.

3 × 13 which is read 3 groups of 13 can be used to solve 13 × 3 since pupils already know that 13 × 3, 3 × 13. (commutative property of multiplication)

N.B. Having gone through some practical activities with pupils they should be encouraged to record their multiplication problems as follow:

Similarly Dienes base ten materials can be used to teach multiplication of bigger number such as 125 × 5.

Thus	125	or	125	
	5		× 5	
	625		500	100 × 5
			100	20 × 5
			25	5 × 5
			625	625

Dear learner try some few examples on your own.

Q73. Describe briefly TWO different ways by which you will lead primary three pupils to solve the problem 15 ÷ 3.

Ans: Two different ways by which 15 ÷ 3 can be explained to pupils are sharing and grouping.

a) SHARING: 15 ÷ 3 can be explained as 3 pupils sharing 15 objects.

►Let pupils pick an object one at a time in turn until all the objects are finished.

►Pupils find out how many objects each pupils get as an answer.

►Pupils write this as 15 ÷ 3 = 5.

b) GROUPING METHOD OF DIVITION

►Take 15 objects.

►Call pupils in front of the class.

►Give each child three objects at a time.

► Ask pupils to mention how many pupils receive the sets of 3 objects.

►Pupils say 5 pupils.

►Guide pupils to write this as 15 ÷ 3 = 5.

Q74. How would you guide primary 6 pupils to solve 15 ÷ 3 using number line.

Ans: � Guide pupils to draw a number line on the floor and mark on it points 1 to 15 of intervals of 1 units starting from zero.

� Guide pupils to step on zero and hop unto 15 on the number line.

� Guide pupils to hop back 3 units until they get to zero.

� Ask pupils to tell how many times the hopping was done to get back to zero.

� Pupils observe this to be 5.

� Thus they write this as 15 ÷ 3 = 5.

Q75. a) which property of operation is shown in each of the mathematical statements.

i) 3 + 5 = 5 + 3 ii) 3 × 5 = 5 × 3

c) Explain each property to a class 2 pupil.

For 3 + 5 = 5 + 3

Ans: i) The property shown above is commutative property.

ii) Explain that whether 3 objects are added to 5 objects OR 5 objects are added to 3 objects the result is the same.

OR

Pupils form set of 3 counters and form another set of 5 counters and put them together to get 8 counters.

They again form a set of 5 counters and another set of 3 counters and then put them together to get 8 counters.

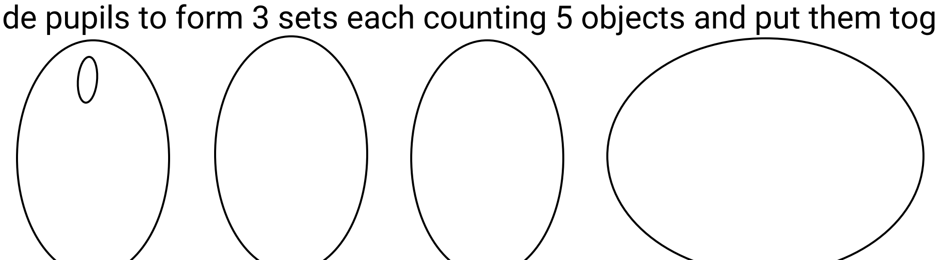
Pupils realize that 3 + 5 = 5 + 3 = 8.

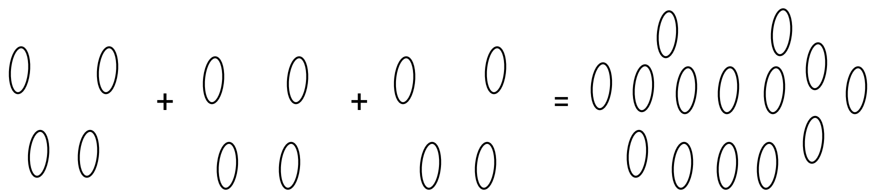
For 3 × 5 = 5 × 3, this property is commutative property of multiplication

Explain that 3 groups of 5 which is read 5 + 5 + 5 = 15 is the same as 5 groups of 3 which is read as 3 + 3 + 3 + 3 + 3 = 15.

OR

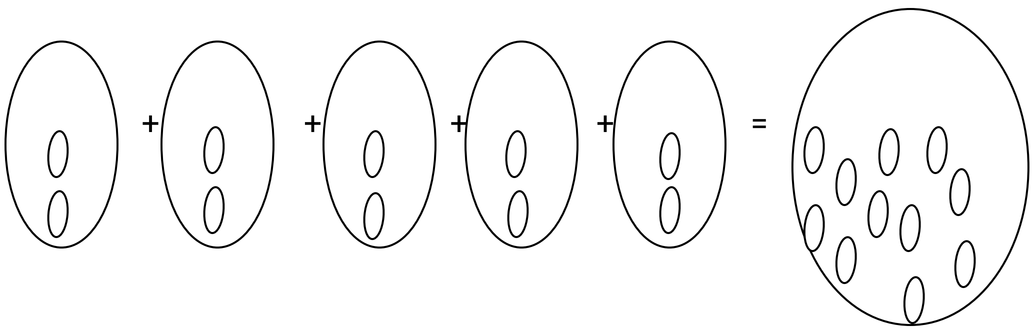
Guide pupils to form 3 sets each counting 5 objects and put them together as follows





Pupils observe them to be 15

Pupils again form another 5 sets each one containing 3 objects as follows



Pupils put them together and observe them to be 15.

Pupils then conclude that 3 × 5 = 5 × 3

Q76. a) Which property of operation is shown in the mathematical statement

3(2 + 4) = (3 × 2) + (3 × 4)

b)Explain to primary 6 pupils.

Ans: a) The property shown is distributive property of multiplication over addition.

b)Explain that when you add 2 and 4 and multiply by 3 you get the same result as when you multiply 3 by 2 and 3 by 4 and add the result.

Q77. a) Which property of operation is shown in the mathematical statement.

3+ (4 + 5) = (3 + 4) + 5

b)Explain this property to a primary 6 pupil.

Ans: a) Associative property of addition

b)Explain that when you first add 4 and 5 and 3 you get the same result as when you first add 3 and 4 and add the result to 5.

N.B. (counters can also be used)

►Form a set of 4 and another set of 5 and put together and add the result to another set of 3.

►Form a set of 3 and another set of 4 and add the result to another set of 5.

► Let pupils compare the result.

►Pupils realize they are equal.

►Thus $3 + (4 + 5) = (3 + 4) + 5$.

Q78. a) Which property of operation is shown in the mathematical statement.

$3 \times (4 \times 5) = (3 \times 4) \times 5$

b)Explain this to a primary six pupil.

Ans: a) Associative property of multiplication

b)Explain that if 4 is multiplied by 5 and the result multiplied by 3, you get the same result as first multiplying 3 by 4 and the result multiplied by 5.

Alternatively.

▬ Let pupils find 4 groups of 5 and then find 3 group of the result.

▬ Pupils get 4 group of 5 as $5 + 5 + 5 + 5 = 20$ then 3 groups of 20 as $20 + 20 + 20 = 60$

Again, pupils find 3 groups of 4 as $4 + 4 + 4 = 12$ and then find 12 groups of 5 as 5 + + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5

▬ Pupils compare the results to realize that they are the same.

▬ Thus, they write $3 \times (4 \times 5) = (3 \times 4) \times 5$.

Q79. How would you guide children in class 1 to identify

a) Solid shapes and their properties.

b) Planes shape and their properties.

Ans: ►Let pupils bring a collection of different kinds of solids such as empty containers like milk tins Milo tins, magi cube, empty soap box, a ludo die, football, pencil etc.

►Let pupils put their collections and sort out different shapes according to their criteria.

►Help pupils to learn the names of the different solids such as:

i) Cuboids which also include cubes.

ii) Cylinders including milk tins, Milo tins and pencils

iii) Prisms which are made up of triangular prisms, pentagonal prism

iv) Spheres which include football etc.

►Let pupils give actual examples of solid shapes such as cuboids, cylinders and prisms.

►Let pupils pick any shape and guide them to identify the faces, edges and the vertices

►Guide children to know that:

a) A face is the flat side (plane) of a solid.

b) An edge is the meeting line of two faces.

c) A vertex is the point two or more edges meet.

►Guide them to investigate the number of faces, edges and vertices for each of the following by completing the table below:

Name of solid	Diagram	No of face	No of Edges	No of vertices
Cuboid		6	12	8

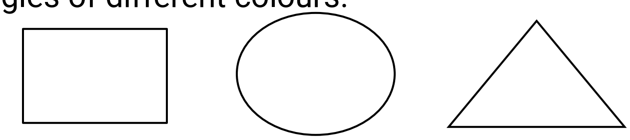
Cube	6	12	8
Triangular pyramid	4	6	4
Rectangular pyramid	5	8	5
Triangular prism	5	9	6
Square pyramid	5	8	5

►Guide children to open an empty solid such as carton and look at the nets.

►Guide them to open other solids and use their nets.

►Make cut out shapes of plane figures such as small circles, triangles and rectangles of different colours.

e.g.



►Help pupils to recognize and name the plane shapes.

►Ask pupils to sort them with the same shape or colours (according to given criteria)

►Let them talk with one another as they work.

►Pick a different shape at a time and ask the children to find a similar one among their collections.

►Ask them to tell how many sides each object has.

►Guide them to draw their shapes. E.g. square, rectangles, triangles.

►Guide children to identify corner of a rectangle or a square as a square corner.

►Guide pupils to test for corners of these shapes by taking a piece of paper and folding into two and refold it to get a square corner.

►Let pupils compare the square corners of the square and the rectangle and triangle.

►Let pupils tell which plane figure has the same square corners as the square corner of the piece of paper.

►Pupils realize this as a square and rectangles.

►Tell pupils that they are called right – angles.

Q80. Describe how you would guide an upper primary pupils to discover the relationship F + V = E + 2. Where F = number of faces, V = number of vertices and E = number of edges.

Ans: ■ Provide pupils with models of a solid shapes such as cube, a triangular prism, cuboids, tetrahedron (triangular pyramid)

- ■ Let pupils count the number of edges, faces and vertices of each model.
- ■ Let pupils use their findings to complete a table as shown below:

Name of solid	No. of edges (E)	No. faces (F)	No. of vertices (V)
Cube	12	6	8
Cuboids	12	6	8
Triangular prism	9	5	6
Tetrahedron	6	4	4

■ Let pupils add the number of faces F to the number of vertices of each model.

■ Ask pupils to compare the sum of vertices (V) and faces (F) with the number of edges.

■ Ask pupils to deduce a relation connecting F, V and E.

■ Pupils discover that F + V = E + 2.

Q81. Many classrooms have a cupboard. List and show how five geometrical concept could be illustrated using this cupboard as an example.

Ans: Geometrical concept illustrated:

At the vertices, points are illustrated.

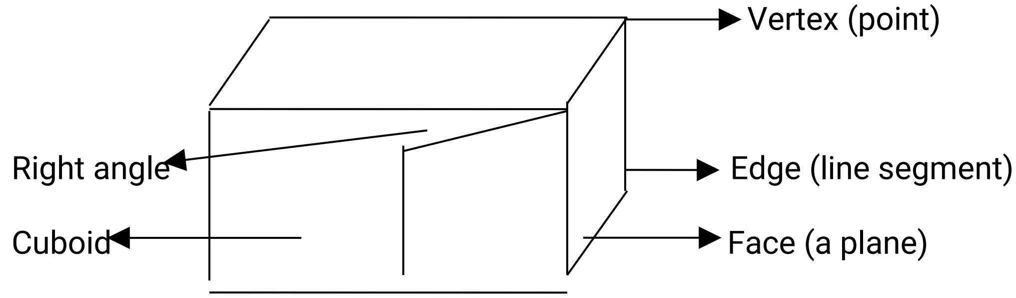
The Faces illustrate planes.

The edges illustrate line segments.

When the door of the cupboard is opened acute angles are formed.

At the corners, right angles are illustrated.

The cupboard as a cuboid is a three dimensional figure (solid shape)



Q82. Explain how you would help a primary six pupils to find two differences between a prism and a pyramid.

Ans: 1) A prism has a uniform cross section but a pyramid has an apex, the slant edge meet at a point called the apex and so has a non – uniform cross section.

2) The side view of a pyramid is always a triangle but the side view of a prism is always a rectangle.

3) Volume of a pyramid is a third of a prism when they have the same base and height.

Q83. a) What do you understand by the term

i) Scheme of work.

ii) Lesson plan.

b)Give reasons.

Ans: a) A scheme of work is the termly plan prepared by the teacher on a weekly basis for topic that are to be covered in the term.

Reason for preparing scheme of work:

i) Help the teacher to determine whether he/she is lagging behind or making progress in the teaching and learning situation.

ii) It also enables a new teacher who takes over from the out going teacher during the course of the term to know where to begin or continue from.

b)A lesson plan is an outline of activities the teacher follows in order to create an effective teaching and learning situation, including determining what to teach, when to teach and for how long it should be taught.

Reason for preparing lesson plan;

i) Guide other teachers to be able to present lesson almost the same way as the class teacher in the absence of teacher.

ii) It enables the teacher to teach with confidence.

iii) For a focused and effective teaching or delivery of the lesson topic or content.

iv) To help identify the relevant basic materials.

v) It gives the teacher a feeling of self – confidence.

Q84. Explain the following terms as used in lesson preparation.

i) Objectives

ii) Relevant previous knowledge (RPK)

iii) Core point

iv) Evaluation

v) Remarks

vi) References

vii) T/L Activities: T/L Materials.

Ans: i) Lesson objectives are statement of what pupils should be able to do or the teacher should be able to achieve by the end of the lesson. It is a statement of the target set. The objectives stated must be measurable, behavioral, observable, specific and achievable and must be time bound.

ii)Relevant previous knowledge (RPK) refers to the knowledge, skills or ideas that children have and which relates to the new thing that is to be learned for which the teacher can use as a starting point and build upon to teach the new lesson.

iii)Core point refers to the main idea that the pupils will learn including the most important knowledge, skills and attitudes that are developed during the lesson delivery.

iv)Evaluation means finding out if the lesson objectives have been achieved. This covers exercises, quizzes (oral or written) giving by teachers to find out the extent to which the knowledge, skills and attitude specified in the objectives have been achieved.

v)Remarks: This is the comment made on the effectiveness of teaching and learning that took place during the lesson and the specific problems or weaknesses observed or encountered during the lesson delivery and which need further action. Usually, reason are given when lesson is not taught. E.g. lesson was not taught due to a rain storm which prevented class from going on.

vi)T/L activities: These are activities which the teacher and the pupils will perform during lesson delivery which should reflect the interaction between the teacher and the pupils, among pupils themselves, and pupils and materials. These activities are described as a step by step manner that gives a picture of what will occur during lesson delivery.

vii)T/L materials: These are materials the teacher and the pupils will use to help understand the various concepts of the lesson. Teachers need to take note on when and how these materials will be used. This is stated in the activities of the lesson plan.

Q85. What is the meaning of S.R.N as used in the lesson plan preparation?

Ans: S.R.N means syllabus reference number e.g. 1.3.2.

1 refers to the year

3 refers to the unit

2 specific objective. Thus 1.3.2 means year 1 unit 3 and specific objective 2

Q86. Explain briefly what is meant by each of the following in the teaching and learning of mathematics.

a) Process objective

b) Affective objective

c) Content objective

d) Behavioral objective

Ans:

a) Process objectives are concerned with the mathematical strategies and processes such as describing, grouping, classifying, skills, ability, generalizing, abstraction, analyzing and problem solving.

b) Affective objectives: This involves the desire to use mathematics in real life situation. It deals with feeling and attitudes one has in mathematics. Affective objective is a continuous development at various levels in the teaching learning process. It may not be achieved in a particular lesson.

c) Content objective: It involves the subject matter and the techniques adopted during lesson delivery.

d) Behavioral objective: It is an objective stated in observable and measurable manner which the teacher will be able to achieve at the end of the lesson. Behavioral objective is thus a written statement of what the children should be able to do by the end of the lesson.

   Performance verbs or action verbs such as add, list, sort, calculate, classify and show are used when stating objectives.

Q87. Describe how you would use concrete materials to help pupils to develop the following number patterns.

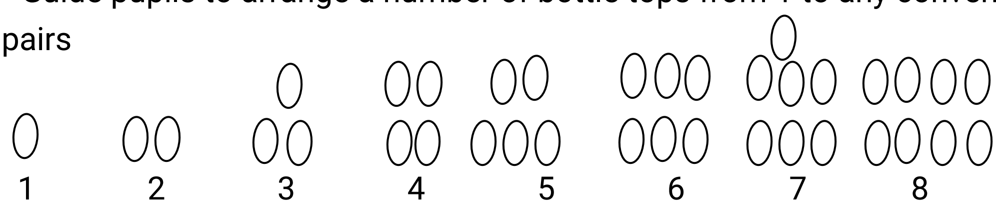
a) Odd and Even numbers.

b) Triangular numbers and square numbers.

c) Prime numbers and composite numbers.

Ans: using bottle tops:

►Guide pupils to arrange a number of bottle tops from 1 to any convenient number in pairs



Let pupils observe the sets and tell what they see.

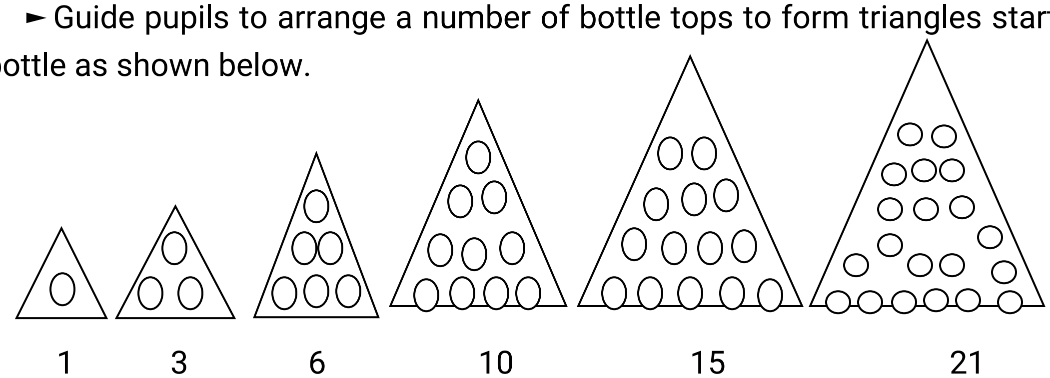
►Pupils observe that some numbers of bottle tops cannot be represented in pairs 1.3.5.7.

►Explain that numbers that are represented in pairs are called even numbers .e.g. 2.4.6.8…..

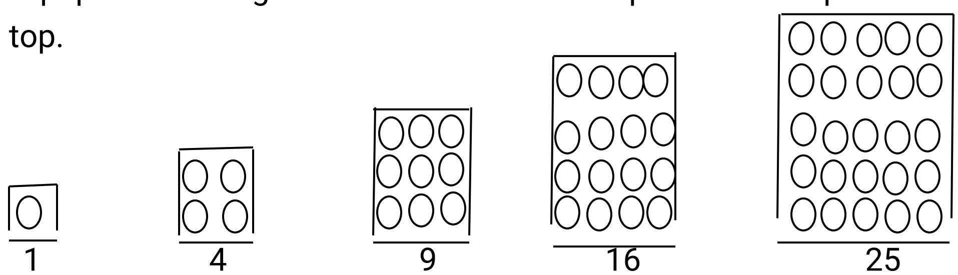
► Explain that numbers which cannot be represented in pairs are called odd numbers

b)Triangular numbers

► Guide pupils to arrange a number of bottle tops to form triangles starting with one bottle as shown below.



►Guide pupils to arrange a number of bottles tops to form squares starting with one bottle top.



►Pupils observe that 1,3,6,10,15,21 and so one bottles tops can be used to form triangles.

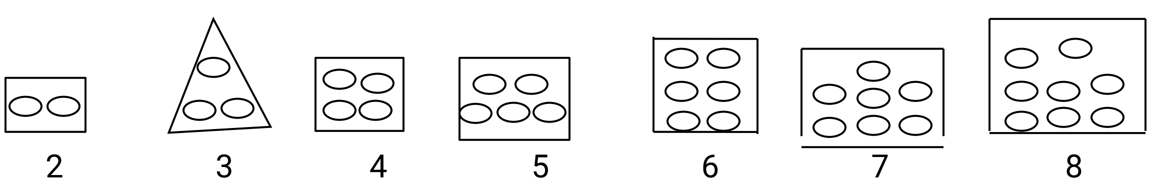
►Explain that numbers that are used to form a triangle are called triangular numbers.

►Pupils infer that numbers that form squares are called square numbers.

►Pupils then list the square numbers as 1,4,9,16,25,36…..

c)for prime and composite numbers:

►Guide pupils to use a number of bottle tops except one to form rectangles in different forms such as below.



► Pupils observe that some number of bottle tops can be used to form rectangles. Explain that number of bottle tops that can be used to form rectangles are called composite numbers pupils list them as 4,6,8,9,10,12……

►Pupils also notice that some number of counters could not form rectangles

►Explain that these are called prime numbers.

►Pupils list the set of prime number as 2,3,5,7,11 etc.

Q88. Explain briefly how you would use sieve of Eratosthene to introduce prime numbers between 1 and 50 to pupils in the upper primary.

Ans: ■ Guide pupils to write all the natural numbers from 1 to 50.

■ Natural numbers from 1 – 50 are as follows

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

■ Let pupils cross out the first number which is neutral (1).

■ Let pupils circle 2 and cross out all the multiples of 2 or divisible by 2.

■ Let pupils circle 3 and cross out the multiples of 3 or numbers divisible by 3

■ Next, circle 5 and cross out all number divisible by 5

■ Next, circle 7 and cross out all numbers divisible by 5

■ Continue till all the numbers have been crossed out or circled

■ Explain that the numbers in the circles represent prime numbers

■ Pupils then write the prime numbers between 1 and 50 as

2,3,5,7,11,13,17,19,23,29,31,37,41,43 and 47

b)■ Guide pupils to list all the natural numbers from 1 to 50

■ Guide pupils to find the set of factors of each natural number.

■ Example; 1 = {1}, 2 = {1,2}, 3 = {1,3}, 4 = {1,2,4}, 5 = {1,5}, 6 = {1,2,3,6}, 7= {1,7}, 8 = {1,2,4,8} 9 = {1,3,9}, 10 = {1,2,5,10},

■ Guide pupils to observe the set of factors of the natural numbers and report their findings.

■ Pupils observe that there are some numbers which have exactly two factors, the number itself and1 whilst others have more than 2 factors.

■ Explain that numbers that have exactly two factors the number itself and 1 are called prime numbers.

■ Explain also that numbers that have more than two factors are called composite numbers.

■ Explain also that numbers that have more than two factors are called composite numbers.

N.B. pupils observe that 1 is not a prime number because 1 has only one factor.

■ Pupils list the set of prime numbers as 2,3,5,7,11,13,17,19,23,29,31,37,41,43 and 47.

Q89. a) What is generic skills?

b)Give <u>TWO</u> generic skills and explain briefly how each is employed in teaching and learning of mathematics.

Ans: Generic skills are the basic skills needed by every child as a foundation upon which other skills and knowledge are built.

Some example of generic skills are, writing, speaking, listening, drawing, observing, investigation, showing, reading.

i) Writing: writing down missing numbers e.g. 1000, 2000, 3000, 5000, 7000.

ii) Reading: reading out expanded form of numerals. E.g. read out 1538 in thousands, hundreds, tens and ones.

iii) Observing: observe another pupils representing a numeral on a number line.

iv) Showing: representing numbers on the abacus and showing to class

v) Speaking: mention the name of a group of objects.

vi) Drawing: draw a group of squares

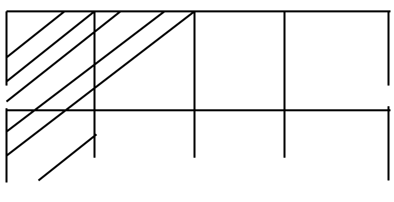
Q90. Describe how you would use a concrete material to introduce to class 2 pupils the concept of $\frac{3}{8}$.

Ans:►Guide children to take a strip of paper as a whole.

►Let them fold it into 8 equal parts and shape 3 parts.

►Explain that the part shaded represent $\frac{3}{8}$

$\frac{3}{8}$



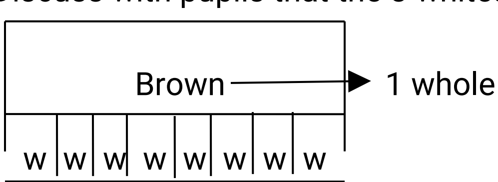
b)Using Cuisenaire rods.

►Let children pick a brown rod which has a value of 8 as a whole.

►Guide children to split the whole into 8 whites.

►Let children select 3 out of the 8whites.

Discuss with pupils that the 3 whites selected represent 3 out of 8 which is written as $\frac{3}{8}$



8 white = 1 brown = 1 whole

Selecting 3 whites out of 8 whites means $\frac{3}{8}$

Q91. Describe briefly one way in which you will introduce the concept "angle" to JHS form one pupils.

Ans: We can introduce the concept of angle as the amount of turning made about a point.

We can make a pupil walk along a line on the floor and then changes his/her direction at a point.

We can also introduce the concept of an angle as the shape of a corner or a turn of a measure about fixed points.

The corner of the walls in the classroom can also be used to introduce the concept of an angle.

A book or cupboard can be opened to introduce an angle.

A pupils can be asked to bend his/her arm around the elbow to introduce the concept of an angle.

Q92. a) Explain what is meant by

1) Direct comparison

2) Indirect comparison

b)Assume that a pupils cannot measure using standard units.

i) Describe how you would guide the pupils to find out which of the following is longer, a pen or a pencil on the teachers table.

ii) Describe clearly how you would guide the pupils to find out which of the following is wider, the length of the classroom and the distance between two poles on the school compound.

Ans: i) Direct comparison involves bringing two or more objects together on the same flat base and comparing the lengths, heights and thickness. E.g. the length of a pen and a pencil can be compared by bringing them on the same flat base and determining which is longer or shorter.

ii)Indirect comparison means comparing objects that cannot be brought on the same flat base.

Example, comparing the length of the classroom and the distance between two poles. In such a situation the same arbitrary units such as strides, hand span, sticks, arms length, thread etc, are used to measure each length and find the number of the arbitrary unit used to cover the length of each object. The greater number of the arbitrary units used is the longer.

b)(i) ►Place the pen and the pencil together with bases on the same flat surface.

►Identify which of them is taller.

►Pupils observe that the taller object is the longer.

(ii) ►Guide pupils to measure the length of the classroom and the distance between the pole using the same arbitrary units.

►Pupils determine the number of arbitrary units which cover the length of the classroom and the number which cover the distance between the poles.

►Pupils determine the number of arbitrary units used to measure each object (i.e. the length of the classroom and the distance between the poles)

►Pupils observe that the length or distance with the greater number of arbitrary unite covering is wider.

Assignment:

Describe how you would guide pupils in the lower primary to find out which of the following is wider;

The width of their classroom door and the width of a window in their classroom. Assume that the pupils cannot measure using units.

Q93. a) Describe the steps you would go through with your pupils in primary class four to introduce them to the measurement of area, including the introduction of the standard unit "the square centimeter" (cm^2).

b)How would you extent this to the measurement of the area of squares and rectangles.

Ans: a) ►Explain area as a measure of the amount of "surface" an object possesses.

►Guide pupils to use different arbitrary units, playing cards or exercise books to measure the surface of a given object. E.g. the measure of their table or desk.

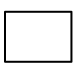
►Pupils notice that they obtain different result for the same surface using different arbitrary units.

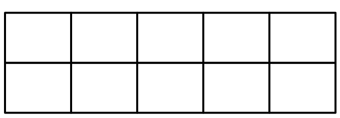
►Explain that since they obtain different results for the same surface, there is the need to use a standard measure so as to get the same result for the same surface of the given objects.

►Introduce the centimeter as the conventional standard unit for the measurement of area.

►Help pupils to cut unit squares of centimeters and use them to cover a given region of squares and rectangles.

►Let pupils label the squares and rectangles as ABCD etc.


Unit square of centimeters


Rectangle A

►Ask pupils to tell how many units squares they used in covering the surface of rectangle A.

►Pupils count the number of units squares of centimeters used to cover the surface of the rectangle.

►Guide pupils to construct squares or rectangle with a given number of square units to demonstrate an initial understanding of the area.

►Ask them to tell how many centimeters squares can cover a given rectangle by counting the number of square units given.

►Guide pupils to establish the relationship between the area, the length and the width of a given rectangle or square by counting the number of squares units along the horizontal (L) level of the rectangle or the square units along the vertical (V) level of the square or the rectangle.

►Guide pupils to multiply the number obtained from the length and that of the width and compare with the number obtained when counted.

►Ask pupils to tell whether they see any pattern.

►Pupils observe that L × B = the total number of square units that covered the surface of the rectangle or square (area)

►Pupils establish that area of square or rectangle = length × width

Area = L × B

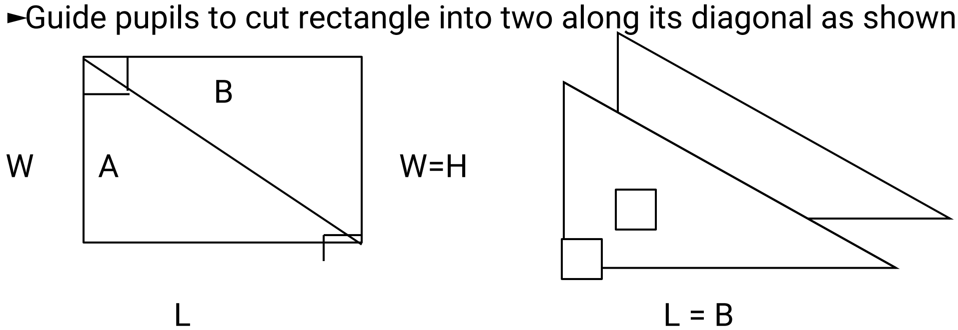
Q94. How would you guide pupils in the upper primary to determine the area of a triangle.

Ans: ►To find the area of a triangle.

►Guide pupils to draw a rectangle for example 8cm by 6cmcm on a manila card.

►Let pupils draw a straight line along the diagonal.

►Guide pupils to cut rectangle into two along its diagonal as shown below;



►Let pupils put together the two right – angled triangles obtained after cutting to determine whether they are the same.

►Pupils will realize they are the same.

►Therefore two right – angled triangles from a rectangle.

►Pupils therefore notice that, finding the area would be the same as half of the total area of the rectangle.

►Hence they write, Area of a Triangle = $\frac{1}{2}$ (Length × Width). But the length of rectangle is equivalent to the base (b) of the triangle and the width is also equivalent to the height (h) perpendicular to the base of the triangle.

►Pupils then write A = $\frac{1}{2}$ bh

Q95. a) What do you understand by the mathematical concept "perimeter"

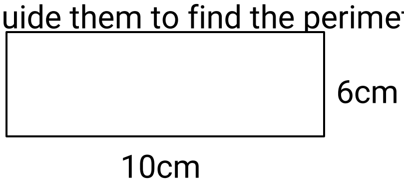
b)Describe how would you introduce the concept of perimeter to a primary six pupils using specific example.

Ans: a) Perimeter is the distance round an object (a geometrical figure)

b)To introduce pupils to the concept of perimeter such as that of a rectangle, pupils may be taken to the school field where they would be made to walk along the edges.

►Let pupils walk around the classroom block or move their hands along the edges of their table or the teachers table. Use a thread to measure all the sides.

►Guide them to find the perimeter of a rectangle below;



Perimeter = 10cm + 6cm + 10cm + 6cm

= 2(10cm + 6cm)

= 2(16cm) = 32cm.

Guide pupils to conclude that the perimeter of a rectangle = 2(length + width)

= 2(L + W)

Q96. How would you guide class six to find out for themselves the difference between a square and a rectangle?

Ans: ▼Guide pupils to use paper cut out of a square and a rectangle to show the following properties in each case.

i) The sides.

ii) Lines of symmetry.

iii) Order of rotational symmetry

iv) Diagonals.

▼Pupils observe that a square has all four sides equal whilst a rectangle has the pair of opposite sides equal.

▼Diagonals of a square are lines of symmetry but the diagonals of a rectangle are not lines of symmetry.

▼Diagonal of a square are at right angles but the diagonals of a rectangle are not at right angles.

▼Order of rotational symmetry of a square is 4 whilst the order of a rotational symmetry of a rectangle is 2.

▼ Lines of symmetry for a square is 4 whilst lines of symmetry for a rectangle is 2.

Q97. Write down SIX skills a child needs to develop to be able to tell time from a regular (analogue) clock.

Ans: ▬ Identify the hour and the minute hand.

▬ Identify the divisions on the clock face.

▬ Tell the time by the hour noting that the minute hand is on the 12 whilst the hour hand is on the number indicating the hour.

▬ Identify the hour that a time is/before/after .e.g. it is five o'clock, it is 10 minutes to three etc.

Q98. a) How would you help pupils in the lower primary to know money?

b)How would you extend this knowledge to be able to know equivalent values of coins and notes.

Ans: ▲Provide sample of all the notes and coins used in Ghana.

▲Let pupils identify them.

▲ Ask pupils to mention the value of a note and coin.

b) ▲To know about money set up a play shop.

▲Ask pupils to bring several different kinds of empty tins and boxes.

▲Now set up a shop in the classroom.

▲Let pupils buy items in the shop and get the correct change.

▲Guide children to exchange notes with coins with their equivalent values.

▲Allow the pupils to use it in writing several different amounts.

Q99. A pupil simplified $\frac{16}{64}$ as follow $\frac{16}{64} = \frac{1}{4}$

a) What is wrong with this approach to the solution?

b) How can you help the pupil to simplify the fraction correctly?

Ans: a) The pupils simply crossed out the digit 6 found in both numerator and the denominator without considering whether 6 is a factor of both 16 and 64 or not.

b)To simplify the fraction, guide pupils to express it as a product of proper fraction i.e. $\frac{16}{64} = \frac{4 \times 4}{4 \times 4 \times 4}$

■ Pupils divide through by 4 to get $\frac{4 \times 4}{4 \times 4 \times} = \frac{1 \times 1}{1 \times 1 \times 4}$

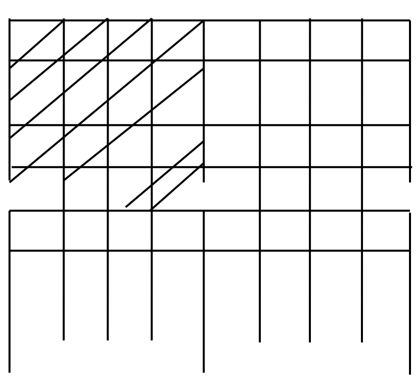
c) In helping pupils to simplify fractions,

1. Let pupils fold a sheet of paper into 64 equal parts and shade 16 out of the 64

2. Ask the child to compare the shaded portion with the total number of parts into which the paper has been divided.

3. Pupils observe that the shaded portion is one – fourth of the total.

4. Thus $\frac{16}{64} = \frac{1}{4}$



$\frac{1}{4}$

Q100. How would you guide upper primary pupils to solve $\frac{2}{5} + \frac{3}{10}$

Ans: ▬ Choose a whole that could be split into tenths. N.B Choose the highest denominator as a whole if it is a multiple of the other denominator.

▬ Pupils therefore choose orange rod and split into 5 red and split into 5 red rods.

▬ Pupils pick 2 red rods to represent $\frac{2}{5}$

▬ Pupils split the orange rod into 10 whites and pick 3whites to represent $\frac{3}{10}$

Diagram:

<table>
<tr><td colspan="10">ORANGE</td></tr>
<tr><td colspan="2">R</td><td colspan="2">R</td><td colspan="2">R</td><td colspan="2">R</td><td colspan="2">R</td></tr>
<tr><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td></tr>
</table>

▬ Pupils are asked to put the 2 red and 3 whites together.

▬ Guide pupils to exchange 2 red rods for 4 whites and add to the 3 whites to get 7 whites.

▬ Pupils compare the 7 whites to 10 whites (1 whole) and get $\frac{7}{10}$ Thus, $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$

Using paper folding and shading.

►Take two rectangular sheets of papers of the same dimensions.

►Fold one into 5 equal parts horizontally and shade 2 parts and refold vertically so that $\frac{2}{5}$ will be equivalent to $\frac{4}{10}$.

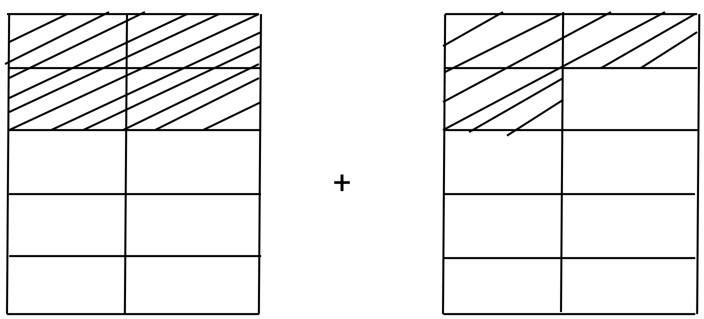
►Fold the other one into 10 equal parts and shade 3 parts for $\frac{3}{10}$

►Add the shaded portions of the two sheets of papers.

►Pupils get 7 shaded portions and compare with the total number of portions into which a whole was divided.

►Thus $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$.

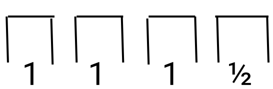
Diagram:



Q101. Describe how you would use concrete materials to explain to pupils that $3\frac{1}{2} = \frac{7}{2}$

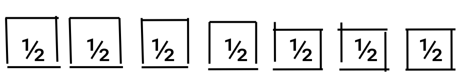
Ans: ►Let pupils take 3 identical rectangular sheets of papers as units or wholes and half of another sheets to represent $3\frac{1}{2}$.

►Diagram



►Let pupils fold each whole into 2 equal parts.

►Diagram



►Let pupils count the number of halves they have altogether.

►Pupils observe that they have 7 halves.

►Thus, they conclude that $3\frac{1}{2} = \frac{7}{2}$

N.B. Cuisenaire rods can also be used. Try it on your own.

Assessment: Describe how you would use concrete material to show pupils that $2\frac{1}{4} = \frac{9}{4}$

Q102. Describe how you would extend the knowledge of addition of fractions to solve $1\frac{3}{5} + 2\frac{1}{3}$

Ans: ■ Let pupils add the wholes and use either paper folding and shading or Cuisenaire rods to add $1\frac{3}{5} + 2\frac{1}{3}$

■ Pupils find the equivalent fraction of $\frac{3}{5}$ and get $\frac{9}{15}$ and $\frac{1}{3}$ and get $\frac{5}{15}$.

■ Pupils put $\frac{9}{15}$ and $\frac{5}{15}$ together to get $\frac{14}{15}$

■ Pupils put the 3 whole and $\frac{14}{15}$ together

■ Pupils then write this as $1 + 2 + \frac{3}{5} + \frac{1}{3}$

$$= 3 + \frac{9}{15} + \frac{5}{15}$$

$$= 3 + \frac{14}{15}$$

$$= 3\frac{14}{15}$$

Q103. a) Write down <u>TWO</u> problems which shows subtraction of fraction you would use to teach upper primary pupils.

b)How would you guide these upper primary pupils to find solution to the word problems stated above.

Ans: a) two word problems which show subtraction of fraction.

i) A woman gives four – sixth liters of her palm oil from her remaining five sixth liters of palm oil to a friend. How many liters of palm oil will be left.

ii) A woman gives half of piece of cloth from her two thirds of cloth. How many will be left.

b)i)Using paper folding and shading.

►Guide pupils to fold a sheet of paper into 6 equal parts and shade five for $\frac{5}{6}$.

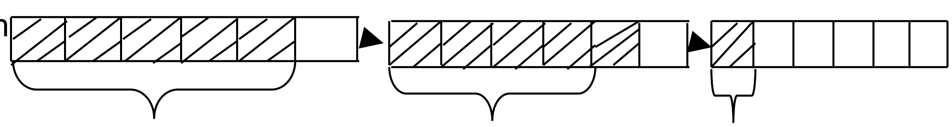
►Ask pupils to take away $\frac{4}{6}$ by unshading 4 parts.

►Ask pupils to tell how many parts shaded left.

►Pupils observe that one part out of six parts shaded left.

►Thus, they write this as $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

Diagram



$\frac{5}{6}$ unshade $\frac{4}{6}$ $\frac{1}{6}$

Algorithm: $\frac{5}{6} - \frac{4}{6} = \frac{5-4}{6} = \frac{1}{6}$

Using Cuisenaire rods:

►Guide pupils to choose a dark green rod as a whole (unit)

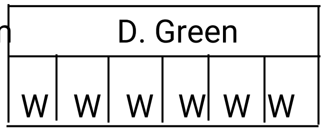
►Let pupils split it into 6 whites rods

►Let them pick 5 whites rods to represent $\frac{5}{6}$

►Pupils take away 4 white rods from the 5 white rods picked leaving 1 white rod.

►Pupils compare the 1 white rod left with the total number of white rods into which the whole was divided.

►Pupils observe this to be $\frac{1}{6}$. Therefore $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

Diagram 

$$\frac{5}{6} - \frac{4}{6} = \frac{5\text{-}4}{6} = \frac{1}{6}$$

Using paper folding and shading to find $\frac{2}{3} - \frac{1}{2}$

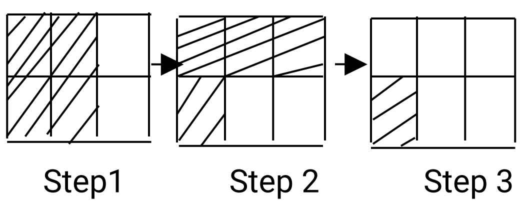
►Guide pupils to fold a sheet of paper into 3 equal parts vertically and shade 2 parts and fold again horizontally into 2 equal parts such that $\frac{2}{3} = \frac{4}{6}$ (equivalent fractions)

►Guide pupils to re-shade and then un-shade half which is represented by $\frac{3}{6}$.

►Ask pupils to tell how many shaded portion left.

►Pupils see this to be one shaded portion which they compare with a whole as $\frac{1}{6}$.

Diagram:



Two thirds - 1half = 4sixth – 3 sixth = 1 sixth = $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

Using Cuisenaire rods.

►Guide pupils to pick a whole that could be split into thirds and halves.

►Pupils choose a dark green rod.

►Pupils split the dark green into 3 red rods and take 2 red for $\frac{2}{3}$.

►Pupils split the dark green into 2 light green rods and take away 1 light green to represent $\frac{1}{2}$.

►Ask pupils to take away 1 light green from 2 red rods.

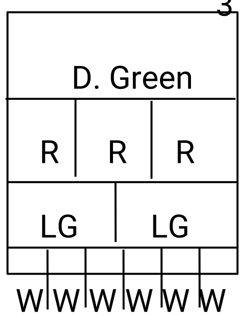
►Since this seems impossible, pupils exchange 2 red rods for 4 whites and the 1 light green for 3 whites.

►Pupils now take away 3 whites from 4 white leaving 1 white.

►Pupils compare 1 white left with the 1 dark green split into 6 whites.

Thus pupils conclude that $\frac{2}{3} - \frac{1}{2} = \frac{4\text{-}3}{6} = \frac{1}{6}$

Diagram:



2R = 4 Whites i.e. $\frac{4W}{6W} - \frac{3W}{6W} = \frac{4W - 3W}{6W} = \frac{1W}{6W} = \frac{1}{6}$

1LG = 3 Whites

Q104. How would you guide pupils in the upper primary to solve the following fractions using co materials: (i) $\frac{1}{2} \times 2$ (ii) $2 \times \frac{1}{2}$ (iii) $\frac{1}{2} \times \frac{1}{3}$ (iv) $5 \times \frac{3}{4}$ (v) $\frac{3}{4} \times 5$ (vi) $\frac{3}{4} \times \frac{3}{5}$.

Ans: Using paper folding and shading:

►Guide pupils to represent $\frac{1}{2} \times 2$ as $\frac{1}{2}$ lots of 2.

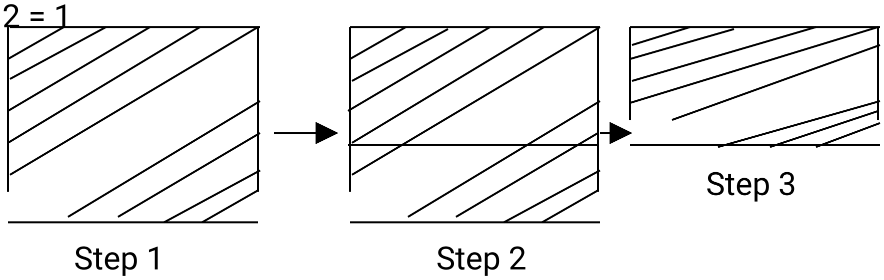
►Ask them to shade two identical rectangular sheets of papers and put them together to represent 1.

►Divide it into two equal parts.

►Pupils observe that one shaded part represent 1.

►Thus, $\frac{1}{2} \times 2 = 1$

Diagram



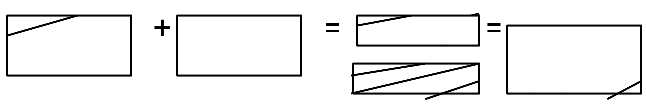
b)►Guide pupils to represent $2 \times \frac{1}{2}$ as 2 lots of $\frac{1}{2}$. i.e. $\frac{1}{2} + \frac{1}{2}$ (repeated addition)

►Shade $\frac{1}{2}$ of each of two identical rectangular sheets of papers.

►Put together the two shaded parts.

►Pupils realize that two halves represent 1.

Thus $2 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$



iii) $\frac{1}{2} \times \frac{1}{3}$

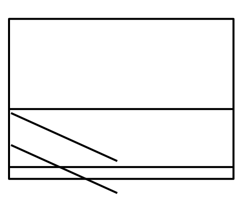
►Let pupils fold a piece of rectangular sheets of paper into 3 equal parts horizontally and shade 1 part for $\frac{1}{3}$.

►Let pupils fold the $\frac{1}{3}$ into 2 equal parts vertically and shade 1 part.

►Explain that the area having the double shading is the answer i.e. one double shaded part out of 6 parts.

►Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Diagram:



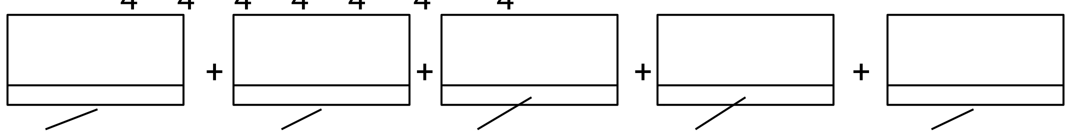
iv) Guide pupils to take 5 identical rectangular sheets of papers.

Guide them to fold each into 4 equal parts and shade 3 parts for $\frac{3}{4}$

Guide pupils to put the shaded portions together as shown below.

Pupils put them together and obtain.

$$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$$



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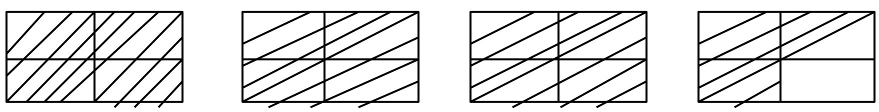
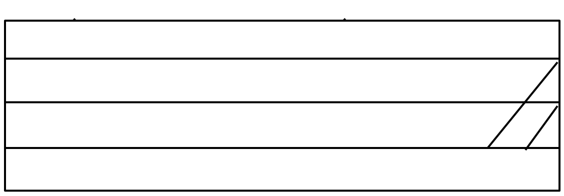


$$1 \quad + \quad 1 \quad + \quad 1 \quad + \quad \frac{3}{4}$$

$$= 3 + \frac{3}{4} = 3\ \frac{3}{4}$$

v) $\frac{3}{4} \times 5$

- Explain that $\frac{3}{4}$ means $\frac{3}{4}$ lots of 5.
- Take 5 identical sheets of papers and put them together to represent 5.
- Fold into 4 equal parts and shade 3 parts.



Pupils count the shaded portions and compare to the whole.

Thus, $\frac{3}{4}\times 5 = \frac{15}{4} = 3\frac{3}{4}$

vi) $\frac{3}{4} \times \frac{3}{5}$

▬ Let pupils take a rectangular sheet of paper.

▬ Divide it into 4 equal parts horizontally.

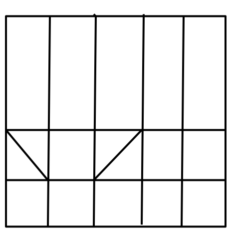
▬ Shade 3 horizontal sections for $\frac{3}{4}$.

▬ Divide the same sheet of paper into 5 equal parts vertically. Shade 3 vertical parts for $\frac{3}{5}$

▬ Explain that the overlapping area of shading gives the answer.

▬ Pupils see that there are 20 portions with 9 portions having overlapping shading.

▬ Hence, pupils write $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

Diagram: 

9 portion out of 20 portions having overlapping shading

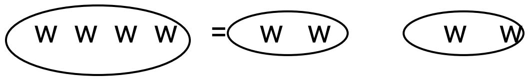
Using Cuisenaire rods

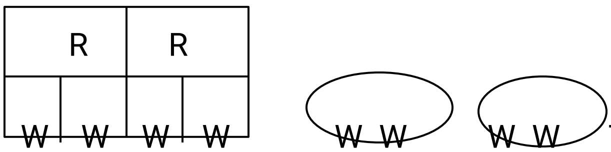
i) For $\frac{1}{2} \times 2$

▲Explain that $\frac{1}{2} \times 2$ means $\frac{1}{2}$ lots of 2.

▲Split each of the 2 red rods into 4 whites.

▲Group the whites into groups of 2 getting 2 groups and pick 1 group (2 whites)

Diagram: 



Thus: $\frac{1}{2} \times 2 = 1$

ii) For $2 \times \frac{1}{2}$

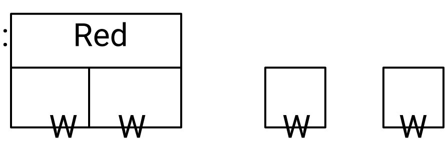
▲Explain that $2 \times \frac{1}{2}$ means 2 lots of $\frac{1}{2}$ i.e. $\frac{1}{2} + \frac{1}{2}$

▲Guide pupils to pick a red rod and split it into 2 whites.

▲Pick 1 white and another 1 white and put together to represent 2 whites.

▲Pupils compare the 2 whites to a whole (red) i.e. 2 whites.

Thus $2 \times \frac{1}{2} = \frac{2}{2} = 1$

Diagram: 

$\frac{2W}{2W} = \frac{2}{2} = 1$

iii) For $\frac{1}{2} \times \frac{1}{3}$

▲Describe the three different ways you would use to help pupils in primary class one to solve the example that $\frac{1}{2} \times \frac{1}{3}$ means $\frac{1}{2}$ lots of $\frac{1}{3}$

▲Guide pupils to pick a whole that could be split into halves and thirds.

▲Pupils split the dark green into 3 red rods as shown below:

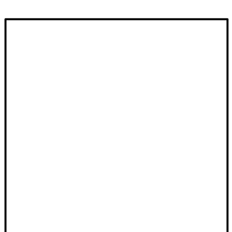
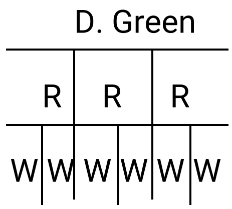


Diagram: 

■ Take 1 red to represent $\frac{1}{3}$.

■ Exchange the red rod for 2 whites.

■ Group the 2 whites into group of 1 and pick 1 group.

■ Pupils pick 1 white (1 group) and compare with a whole (6 whites)

■ Pupils observe that $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

N.B. Use several examples to develop the standard algorithm; thus, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

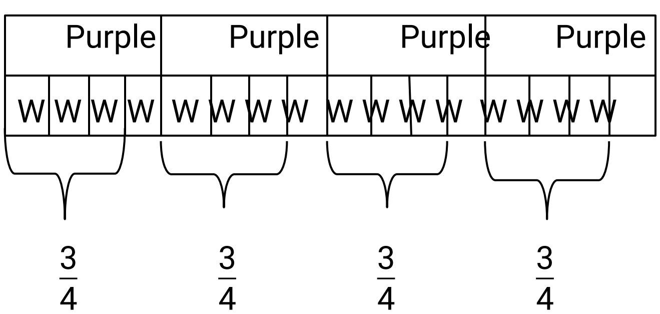
iv) $5 \times \frac{3}{4}$

■ Explain that $5 \times \frac{3}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

■ Guide pupils to choose a whole that could be split into fourths.

■ Pupils choose a purple.

■ Pupils split the purple into whites and 3 whites from each of the 5 purples are put together .

Diagram: 

15 whites compared to a whole is $\frac{15}{4}$, thus $5 \times \frac{3}{4} = \frac{15}{4}$

v)For $\frac{3}{4} \times 5$ means $\frac{3}{4}$ groups of 5.

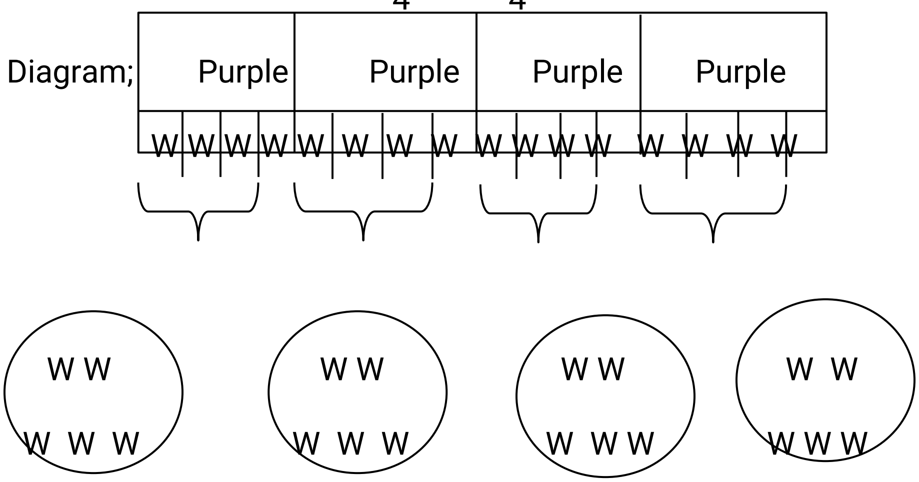
■ Split each of the 5 purple rods into fourths to get a total of 20 whites.

■ Group the 20 whites into group of 5getting 4 groups and picking 3 groups.

■ Pupils pick 15 whites to represent $\frac{3}{4}$ of 20.

■ Thus pupils write this as $\frac{3}{4} \times 5 = \frac{15}{4}$

Diagram;



Picking 3 groups of 5 = 15 whites compared with a whole (1 purple) = $\frac{15}{4}$

vi)for $\frac{3}{4} \times \frac{3}{5}$

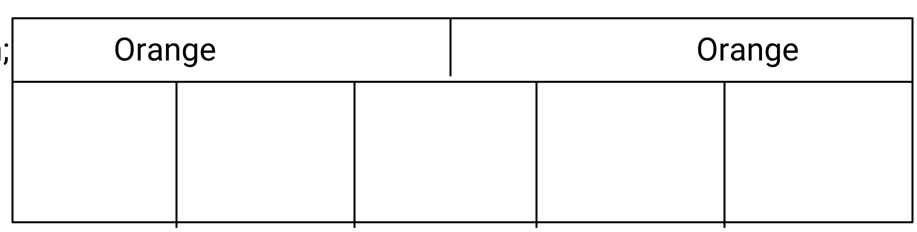
■ Explain that $\frac{3}{4} \times \frac{3}{5}$ means $\frac{3}{4}$ lots of $\frac{3}{5}$

■ Guide pupils to choose a whole that could be split into fourths and fifths.

■ Pupils choose 2 orange rods.

■ Pupils lay the 2 orange rods as follows.

Diagram;



Purple Purple Purple Purple Purple

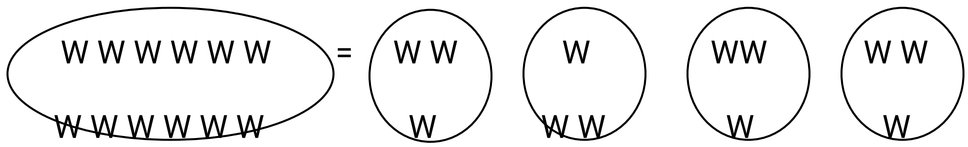
W W W W W W W W W W W W W W W W W W W W

■ Guide pupils to split the whole into fifths (5 purples) and pick 3 purples for $\frac{3}{5}$

■ Guide pupils to exchange the 3 purple for 12 whites.

■ Guide pupils to group the 12 whites into 4 groups of 3 and pick 3 groups.

Diagram;



3 groups of 4 picked represent 9 whites out of 20 whites.

Thus, $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

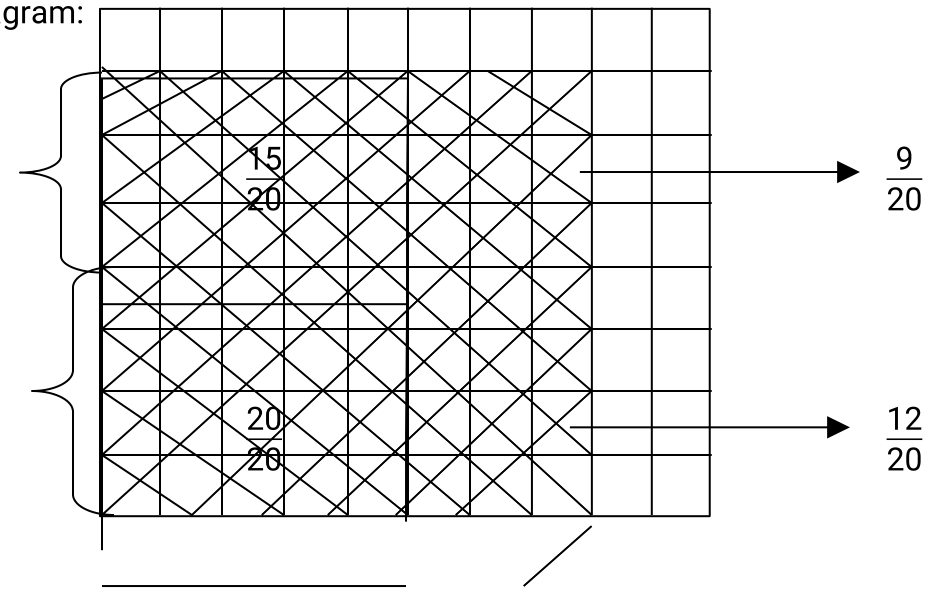
Q105. Explain how you would use diagrams or concrete material to solve $1\frac{3}{5} \times 1\frac{3}{4}$ with primary pupils.

Ans: ■ Divide sheets of paper into fifths and shade $1\frac{3}{5}$ vertically.

■ Fold the same sheet of paper into fourths and shade $1\frac{3}{4}$ horizontally to overlap.

■ The total overlapping area gives the following answer:

Diagram:



■ Pupils add the portions with overlapping shading.

■ Thus $\frac{20}{20} + \frac{15}{20} + \frac{9}{20} + \frac{12}{20} + \frac{56}{20} = 2\frac{4}{5}$

ASSIGNMENT:

1) Describe how you would guide upper primary pupils to solve the following using concrete materials. (i) $3 \times \frac{1}{4}$ (ii) $2\frac{1}{4} \times 1\frac{3}{4}$

N.B. Students can also be asked to write word problems before using concrete materials to solve it.

Example: write a word problem which shows a multiplication of fraction.

What is the cost of three – fourth liters of kerosene if 1 liter cost half pesewa. This is $\frac{3}{4} \times \frac{1}{2}$. Then, you use either paper folding or Cuisenaire rods to solve it.

To understand the concept of division of fractions better, it is best to develop the concept using word problem from the start. Example ($\frac{1}{4} \div 2$). Dividing a fraction by a whole. This could be acted in a word from as follows:

(a) Dividing fractions by a whole.

Q106. Ama went to the market and bought a $\frac{1}{4}$ piece of cloth and shared between her 2 daughters i.e. $\frac{1}{4} \div 2$. Describe how you will help a pupil in class 6 to solve this using concrete materials.

Ans: Using paper folding:

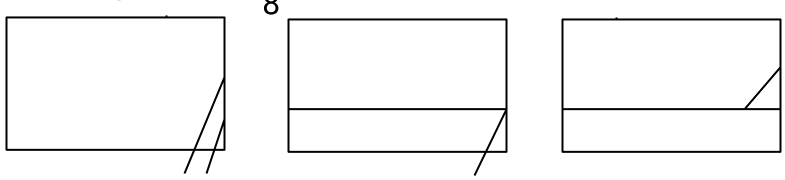
▲Fold a rectangular sheet of paper into (4) equal parts and shade one part.

▲Divide into two equal parts horizontally.

▲Ask pupils to take 1 part.

▲Pupils observe 1 part to be $\frac{1}{8}$

Diagram:



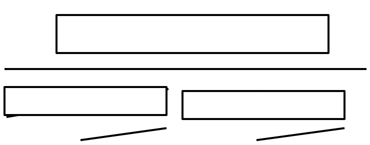
Step 1 Step 2 Step 3

Developing algorithm: $\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

Alternative solution.

$\frac{1}{4} \div 2$ can be written as $\frac{¼}{2}$

Using diagram, this can be represented as:



$= \frac{\frac{1}{4}}{2}$

Procedure:

■ Take a whole and divide it into 4 equal parts and shade 1 part (i.e. the portion the fraction represent as shown in the diagram.)

■ Since the denominator is 2, take2 wholes of the object (it could be a rectangular sheet and divide into four equal parts) i.e. into part that the denominator is divided and shade all.

■ Count the number of halves in the diagram

■ Write that as the numerator over the denominator.

Using Cuisenaire rods

►Explain that $\frac{1}{4} \div 2 = \frac{\frac{1}{4}}{2}$

►Take purple rod and split it into four, i.e. into 4 white rods and pick 1 as the numerator.

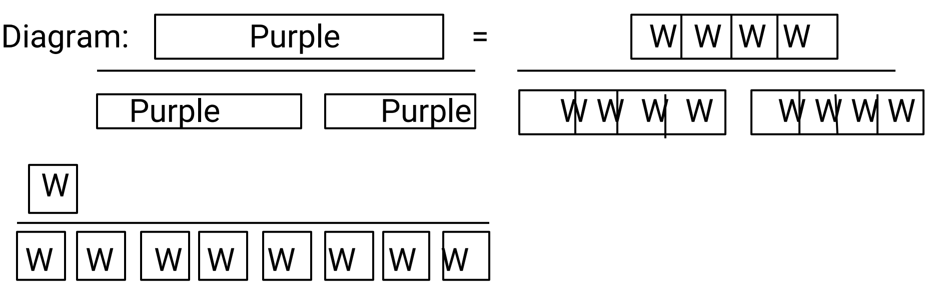
►Since the denominator is 2, take 2 purple rods and split them into 8 white rods as in the diagram below.

►Pupils find that the numerator is 1 white out of 8 whites.

►Thus, they observe that

$$\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$

Diagram:



Let us try another example.

Q107. How would you describe this: $\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Ans: Procedures

►Fold a whole into 3 equal parts and shade 2 parts.

►since the denominator is 3, take 3 wholes and fold each whole into 3 equal parts and shade all to get 9.

►Count the number of thirds in the diagram and write that of the numerator over the denominator.

►Thus, $\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Thus, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Lets us consider another example;

Q108. [$4 \div \frac{1}{2}$] (dividing a whole by a fraction) in a word problem. This can be explained as how many $\frac{1}{2}$ liters full bottle of kerosene can you get from 4 liters of kerosene.

Ans: ■ Explain that $4 \div \frac{1}{2} = \frac{4}{½}$

■ Guide pupils to take 4 whole and divide into 2 equal parts and shade all to get 8 shaded portions for the numerator.

■ Take another sheet of paper and fold into 2 equal parts and shade 1 for $\frac{1}{2}$ for the denominator

■ Pupil observe that there are 8 halves in the numerator and 1 half in the denominator.

■ Thus, they write this as $4 \div \frac{1}{2} = \frac{4}{1} \times \frac{2}{1} = \frac{8}{1} = 8$

ASSIGNMENT:

Work out the following using paper folding or Cuisenaire rods:

a) $3 \div \frac{1}{4}$ b) $\frac{1}{4} \div 3$

Q109. How would you guide an upper primary pupils to act out $\frac{1}{4} \div \frac{1}{2}$ using word problem?. How would you solve this problem?.

Ans: How many $\frac{1}{2}$ liter full of oil can you get from $\frac{1}{4}$ liter full of oil?

►Explain that $\frac{1}{4} \div \frac{1}{2} = \dfrac{\frac{1}{4}}{\frac{1}{2}}$

►Pupils fold a whole into 4 equal parts vertically and shade 1 part and fold again horizontally into 2 equal parts for the numerator.

►Pupils pick another whole of the same dimension, fold into 2 equal parts vertically and shade 1 and refold horizontally into 4 equal parts for the denominator.

►Pupils count the number of shaded portions and write that of the numerator over the denominator

►Pupils observe that to be 2 shaded portion out of 4.

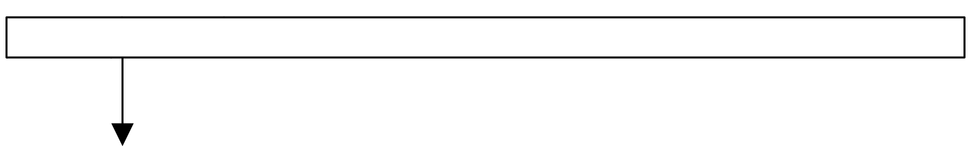
►Thus, $\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$

► $\frac{1}{4} \circ \frac{1}{2} = \frac{1/4}{1/2} = \frac{1}{2}$

Development of concept of decimal fraction.

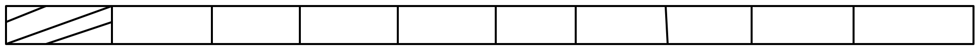
Q110. Consider a strip of paper as a unit. Divide the piece of paper into four equal parts and shade one.

Ans:



This is one quarter or $\frac{1}{4}$

Also take a strip of paper and divide it into ten equal parts and shade one.



This is one tenth $\frac{1}{10}$

This could also be called zero point one as 0.1, that is the decimal name of $\frac{1}{10}$

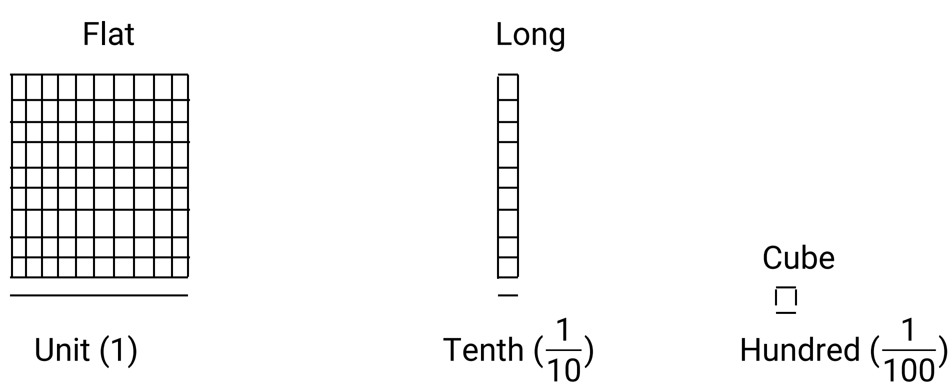
Using a meter rule, 1 cm could be divided into ten equal parts and one part represents 1 mm.

One tenth (or $\frac{1}{10}$) of 1 cm could be written as 0. 1 cm

Q111. Consider a flat of multi – base block as a unit. There are ten longs in a flat. therefore taking one long means $\frac{1}{10}$ of a flat. That is 0.1of a flat.

If for instance, a flat is accepted as the unit, which is one (1),then a long is equivalent to one tenth ($\frac{1}{10}$) and the cube is also equivalent to one hundredth($\frac{1}{100}$).

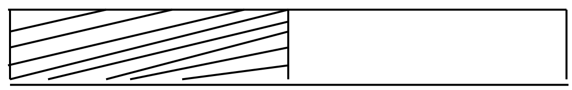
Refer to the diagram below.



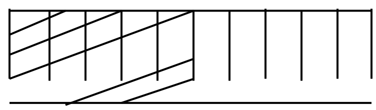
If 2 longs are taken, they represent $\frac{2}{10}$ or 0.2 of the flat.

Converting common fractions and vice versa.

(1) Consider $\frac{1}{2}$ of the diagram below.

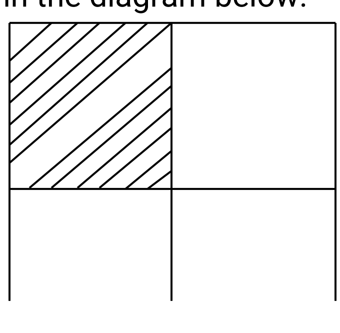


If the entire unit is divided into 10 equal parts, then the shaded portion represents $\frac{5}{10}$

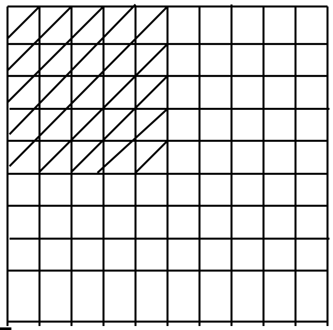


$\frac{5}{10}$ is 0.5. therefore $\frac{1}{2}$ = 0.5

(2) Consider $\frac{1}{4}$ in the diagram below.



Divide the entire unit into 100 equal parts.



This shows $\frac{1}{4} = \frac{25}{100}$

Since $\frac{25}{100}$ is 0.25, therefore $\frac{1}{4}$ = 0.25.

Once the concept has been acquired using teaching/learning materials, Multi – base blocks. 100 square board, etc, algorithm can be developed as shown below.

(1) $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

Since $\frac{5}{10}$ means five tenths which is 0.5, $\frac{1}{2}$ = 0.5

(2) $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$

Since $\frac{4}{10}$ means four tenths which is 0.4, $\frac{2}{5}$ = 0.4

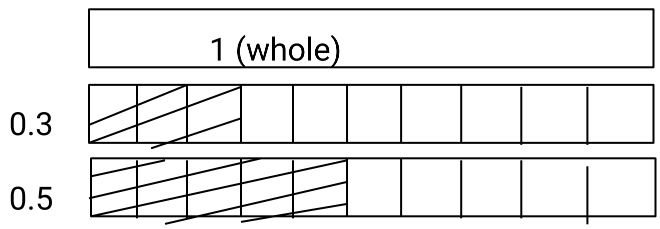
(3) $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$

Since $\frac{25}{100}$ means twenty – five hundredths which is 0.25, $\frac{1}{4}$ = 0.25

Comparing decimal fractions

Compare 0.3 and 0.5

We can compare to see which is bigger using fractional board

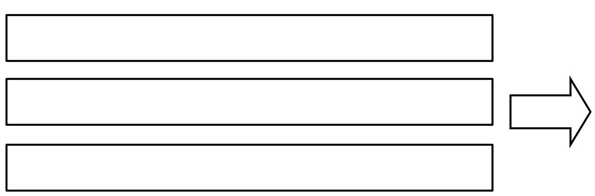


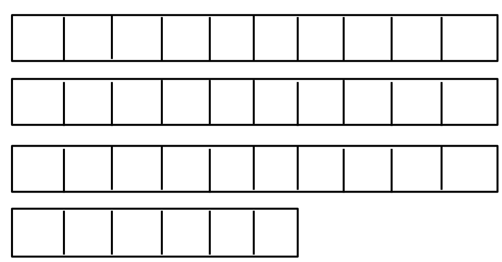
Therefore 0.3 < 0.5 or 0.5 > 0.3

Compare 0.3 and 0.06

(a) Using Multi – based Blocks

0.3 is represented by 3 longs 3 longs is represented by 30 cube





While 0.06 is represented by 6 cubes

Changing all into cubes, 0.3 will give us 30 cubes, while 0.06 is 6 cubes . therefore 0.3 > 0.06.

(b) Using equivalent fractions

0.3 is three tenths ($\frac{3}{10}$) and 0.06 is six hundredths ($\frac{6}{100}$),

0.3 would be $\frac{30}{100}$. = $\frac{6}{100} < \frac{30}{100}$

Therefore $0.06 < 0.3$

Ordering decimal fractions

Order the following decimals; 0.5, 0.25 and 0.08.

Using the idea of equivalent fractions,

$$0.5 = \frac{5}{10} = \frac{5 \times 10}{10 \times 10} = \frac{50}{100}$$

$$0.25 = \frac{25}{100}$$

$$0.08 = \frac{8}{100}$$

Therefore $\frac{8}{100} < \frac{25}{100} < \frac{50}{100}$

$0.08 < 0.25 < 0.5$

(b)Using place values

	One	Tenths	Hundredths
	1	3	5
+	2	2	3
	3	5	8

Find the sum

(1) 0.5 + 2.4 (2) 2.63 + 4.24 (3) 4.57 + 3.12

subtraction of decimals

Find 0.534 – 0.213

(a) Using place values

Ones		Tenths	Hundredths	Thousandths
0	.	5	3	4
- 0	.	2	1	3
0	.	3	2	1

(b)Using short form

$$\begin{array}{r} 0.534 \\ -\ 0.213 \\ \hline 0.321 \end{array}$$

Find;

(1) $\begin{array}{r} 1.025 \\ -0.023 \\ \hline \end{array}$

(2) $\begin{array}{r} 1.364 \\ -1.258 \\ \hline \end{array}$

(3) $\begin{array}{r} 0.362 \\ -0.247 \\ \hline \end{array}$

Multiplication

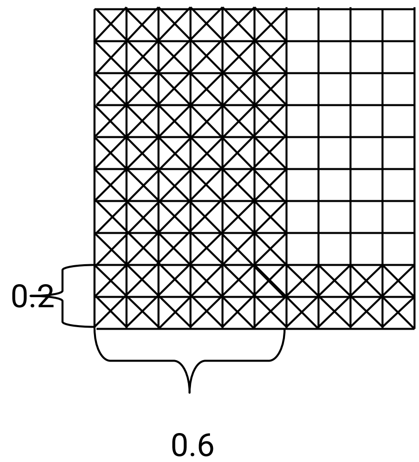
Multiplication by a whole number

Find 0.146 × 3.

Step 1	3 × 6 thousandths	=		1 hundredth	8 thousandths
Step 2	3 × 4 hundredths	=	1 tenth	2 hundredths	
Step 3	3 × 1 tenth	=	3 tenths		
Step 4	3 × 0 one	=	0 one		

Now. (1) calculate 0.6 × 0.2, using a square paper.

Practically a square paper (10 by 10) or graph sheet can be used to model this problem. One side of the square paper (10 by 10) should be represented by 0.6 meaning 6 out of 10 square and shade accordingly. The other side of the square paper should be represented by 0.2, meaning 2 out of 10 spaces and shaded accordingly, this time round in an opposite direction. Find the double shaded (overlapping) area as a fraction of the unit which is 100 space as shown below.



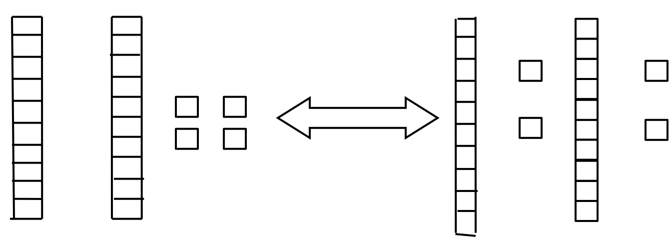
The area with double shading represent 12 out of 100 spaces which is equal to $\frac{12}{100}$ = 0.12.

Division of decimal fractions

(1) division of a decimal fraction by a whole number

(1) find 0.24 ÷ 2

0.24 could be represented by 2 longs and 4 cubes.



2 tenths ÷ 2 = 1 tenth

4 hundredths ÷ 2 = 2 hundredths

Therefore 0.24 ÷ 2 = 0.12

Collecting and handling data

Introduction to statistics

The word statistics has two different but related meanings. In the most common usage, statistics means "a collection of numerical data" for example, we could look at the statistics that show the population of towns and villages in a particular region.

The word statistics also refers to a branch of mathematics that deal with analysis of statistical data. The two branches are descriptive and inferential.

1) Collecting data

Data can be obtain from experiments, studies, surveys, records, observation and or participation, interviews, as well as other areas of research. In the classroom situation it is the responsibility of the teacher to make data collection as practical as possible. Pupils/students can be made to carry out a simple survey to collect data by using some or all of the following:

►Days of the week pupils/student were born.

►The number of different types of bottle tops.

►Heights of pupils/student in a class.

►Favorite food of pupils/students.

►Monthly births at a given hospital.

►Rainfall patterns throughout the year.

►Weekly attendance of pupils/students in a class.

►Marks scored by pupils/students in a mathematics test.

2) Organizing data

One way of organizing data is a frequency table.

A frequency table is a table containing item in an observed data and their corresponding frequencies.

It could be heights, weight, ages of pupils/students or marks scored by pupils/students in a class.

Activity 1

The shoe size of pupils in a class were measured by pupils as follows,

35,37,37,35,36,36,36,35,38,38,38,

39,38,36,36,36,40,40,37,37,37,37,37,

37,41,42,37,37,37,37,37,38,38,38,38,

Put this data on a frequency table.

Table 1A frequency distribution table.

Shoe size	Tally	Number of pupils
35	///	3
36	//// /	6
37	//// //// //	12
38	//// ///	8
39	/	1
40	//	2
42	/	1
42	/	1

3) Representing data

Data can be representing by diagrams for easy interpretation. of data. They are pictogram.

Line graph, Block graph, Bar chart, Pie chart and others.

1. Pictogram

This is a pictorial representation of data which uses pictures, symbols and/or diagrams to represent a specific unit. Pictogram uses a key.

Activity 2

20 students in a class mentioned particular days of the week on which they were born

The result is shown in Table 2.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Students born	2	2	1	4	5	3	3

2. Block graph

A simple square or rectangular block is used to represent the units. The units are built with concrete materials like matchboxes. Cuisenaire rods, multi – base block [cubes] etc. Block graph does not have a vertical axis but a horizontal axis. The horizontal axis represents the items. The space between the adjacent blocks must be at the same width and the block themselves must be of the same width. Like that of the pictogram ,it has a key.

Activity 3

Draw a block to represent data in table 2 of activity 2.

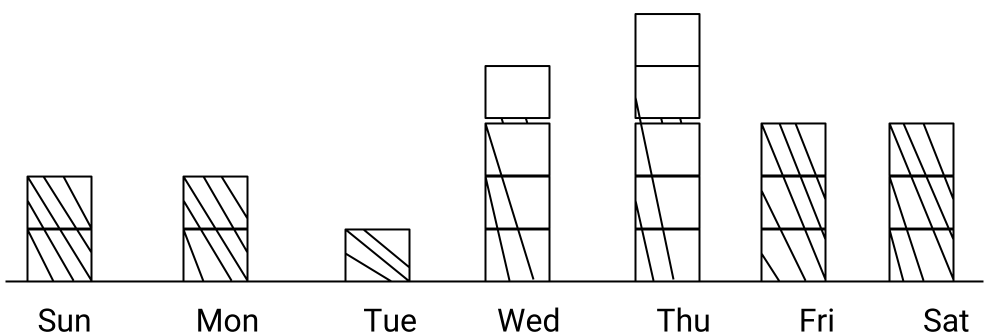


Figure 1 Days of the week on which students were born

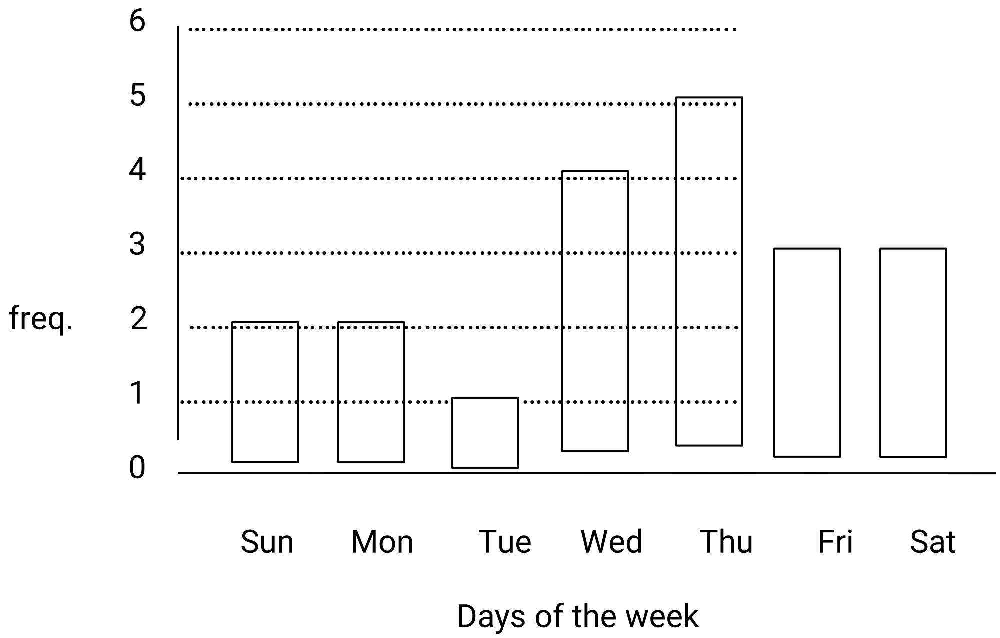
3. Bar graph

In drawing any of the bar graph.

1. The distance between any two adjacent bars must be equal.
2. The width of bars must be the same.
3. The height of each bar is proportional to the number of item in the column.

Draw a bar graph to represent data in table 2 of Activity 2.

Students born on a particular day of the week



Stem – and leaf plot

Numerical data can be put into a stem – and – leaf plot

Construct a stem – and – leaf plot using the data below

57, 13, 24, 17, 13, 18, 12, 42, 39, 35,

11, 55, 55, 44, 48, 36, 19, 30, 19, 22,

Table 6

Stem	Leaf

1	3,7,3,8,2,1,9,9
2	4,2
3	9,5,6,0
4	2,4,8
5	7,5,5

Finding averages (mode, median and mean)

(1) Mode

Mode is the item which occurs most. In terms of frequency it is the term with the highest frequency one can find it using a frequency table

Find the mode of the following distribution

Table 7

Shoe size	No. of pupils
35	3
36	6
37	12
38	8
39	1
40	2
41	1
42	1

From Table 7, the modal shoe size is 37 since it occurs most frequent with as many as 12 pupils having shoe size of 37.

The stem – and leaf plot in Table 8 shows the distribution of males' ages in a town. Find the modal age of the distribution

Table 8	Stem	Leaf
	1	7,6,6,5
	2	1,3,6,9,4,2,8,6,5,2,7,6,4,4,6,6
	3	5,1,3,4,5,5,7,6,7,5,6
	4	4,5,1.

The modal age is 26 years.

(2) Median

Median is the middle item or number when data is arranged in order of increasing or decreasing magnitude.

(1) Find the median of the given data of ages in years of 13 children in a group

4, 5, 6, 6, 6, 6, 6, 7, 7, 7, 8, 8, 9,

The total number of data is 13, therefore the middle item is 7^{th} ,i.e. 6 years.

Hence the median is 6 years.

(2) Find the median number of births at a given hospital using the data below

5, 6, 7, 2, 1, 3, 9, 5, 6, 7, 8, 9, 6, 4,

First we re – arrange the numbers as follows:

Either 1, 2, 3, 4, 5, 5, 6, 6, 6, 7, 7, 8, 9, 9 (in ascending order)

Or 9, 9, 8, 7, 7, 6, 6, 6, 5, 5, 4, 3, 2, 1 (in descending order)

In this distribution the total number is 14, therefore the median is the arithmetic mean of the middle pair (arranged), which is in this case $(\frac{7th+8th}{2}) = \frac{6+6}{2} = 6$

Therefore, the median birth number is 6

Note:

1. When the total number is odd, the median is the middle number.

2. When the total number is even, the median is the arithmetic mean of the middle pair

From the stem – and – leaf plot, find the median mark scored in a mathematics test using the scores below:

17, 21, 44, 35, 28, 31, 26, 33, 34, 29, 35, 16,

24, 37, 22, 45, 27, 26, 41, 36, 16, 25, 22, 35,

37, 27, 26, 15, 24, 28, 26, 35, 36, 26.

First we plot the stem – and leaf without re – arrangement.

Table 9

Stem	Leaf
1	7, 6, 6, 5
2	1, 8, 6, 9, 4, 2, 7, 6, 5, 2, 7, 6, 4, 8, 6, 6
3	5, 1, 3, 4, 5, 5, 7, 6, 7, 5, 6
4	4, 5, 1

Then re- arrange the marks in the leaf part of the stem – and – leaf plot in ascending order.

Table 10

Stem	Leaf
1	7, 6, 6, 5
2	1, 8, 6, 9, 4, 2, 7, 6, 5, 2, 7, 6, 4, 8, 6, 6
3	1, 3, 4, 5, 5, 5, 5, 6, 6, 7, 7
4	1, 4, 5

Note that as regards the example in Activity 9, since the total number is 34 (even), the median will be the arithmetic mean of the 17^{th} and the 18^{th} marks, 27 and 28 respectively.

i.e. $\left(\frac{17th+18th}{2}\right) = \frac{27+28}{2} = 27.5$

hence, the median mark scored is 27.5.

(3) Mean

Mean is the same as arithmetic average. In calculating the mean:

1. Find the sum of the numbers in a given data.
2. Find the total number of individual items in the data.
3. Divide the sum by the total number.

The weight in kg of children in a group are as follows:

11, 23, 34, 45, 52, 33, 44 and 10

What is the mean age in kg?

We first find the sum of their ages.

Sum = 11 + 23 + 34 + 45 + 52 + 33 + 44 + 10 = 252

Mean = $\frac{252}{8}$ = 31.5 (kg)

29, 23, 14, 23, 17, 37, 14, 14, 23, 18

12, 35, 31, 29, 14, 23, 31, 23, 35, 19

31, 29, 23, 14, 12, 24, 23, 24, 29, 31

31, 24, 43, 29, 24, 25, 37, 19, 24, 14

(1) Construct a grouped frequency table, using the interval 12 – 16, 17 – 21,

Table 12

Mark	Tally	Frequency (f)	Class mid point (x)	fx
12 – 16	~~////~~ ~~////~~	10	14	140
17 – 21	~~////~~ /	6	19	114
22 – 26	~~////~~ ~~////~~ ////	14	24	336
27 – 31	~~////~~ ~~////~~ ///	13	29	377
32 – 36	///	3	34	102
37 – 41	//	2	39	78
42 - 46	//	2	44	88

(2) Find the modal class and the mean of the distribution.

From Table 12, since the class with the highest frequency, i.e. 14, is 22- 26,

Modal class is 22 – 26.

To find the mean of the distribution, Total frequency = 50 and Total of fx = 1235.

Hence mean = $\frac{1235}{50}$ = 24.7

With knowledge in statistics so far, let us look at the following DBE examination questions:

Q1. The following is the frequency distribution table showing the marks obtained by 20 pupils in a class test.

Marks	1	2	3	4	5	6
frequency	2	3	5	4	4	2

Describe the steps you would take class six pupils through to represent the above information as a bar chart.

Ans: ►Guide the pupils through the following steps by asking pupils to draw two perpendicular axes on a graph sheet.

►Label the horizontal axis "Days of the week" and the vertical axis "number of pupils" or frequency.

►Calibrate the frequency axis taken into account the highest frequency.

►Mark the width of the bars and write out a day of the week for each.

►Construct a rectangle (bar) for each day with height equal to its frequency marked on the frequency axis.

►Spaces between each bar must be equal.

►Give the graph a Title.

►Draw the graph

Q2. The marks obtained by 20 pupils in a class exercise are as follows:

1 6 2 5 2 3 3 4 4 5

4 3 2 5 3 1 5 4 3 6

Describe how you would guide primary six pupils to represent the above information in a simple frequency table and represent it diagrammatically as a bar chart.

Ans:

i) Ask pupils to draw a table and write column heading: tally and frequency.

ii) Lead pupils to identify the least mark 1 and the highest mark 6.

iii) Let pupils list the marks from 1 to 6 under the column labelled mark.

iv) Pupils mark a tally or stroke for each mark under column labelled tally.

v) Pupils count the strokes made in the column tally for each mark and record it as frequency in the column headed frequency.

vi) Let pupils give a little to the table.

Below is the frequency distribution table;

Marks	Tally	Frequency
1	//	2
2	///	3
3	~~////~~	5
4	////	4
5	////	4
6	//	2
Total		20

Q3. Using the set of numbers 2, 7, 6, 5, 9, 3, 2, 5, 2. Explain clearly each of the following concept to primary six pupils.

i) Mode ii) Median iii) Mean

b)Show and explain how you will guide primary 6 pupils to represent the data below using stem and leaf plot.

12	23	14	35	29	25
34	31	16	27	25	16
26	25	19	20	21	18

Ans: i) Explain that the mode is the number that occur most.

- Pupils look into the set of numbers and realize that 2 is the mode.
- Thus, mode in 2, 7, 6, 5, 9, 3, 2, 5, 2 is 2.

ii) Explain median as the middle item when the distribution is arranged in order of magnitude (ascending or descending)

- Pupils arrange the given numbers in order of magnitude as shown below; 2, 2, 2, 3, 5, 5, 6, 7, 9, 9
- Pupils select the two middle numbers.
- Pupils find the average of the two numbers as $\frac{5+5}{2} = \frac{10}{2} = 5$
- Pupils note that the median is 5.

iii) Pupils determine the mean by adding all numbers given and dividing by the number of items in the distribution (number of addends)

Mean $\frac{9 + 2 + 7 + 6 + 5 + 9 + 3 + 2 + 5 + 2}{10} \quad \frac{50}{10} = 5$

- Pupils note that the mean is 5.

b)Pupils arrange the numbers in ascending order and by tens.

i.e. 12 14 16 16 18 19 20 21 23

25 25 25 26 27 29 31 34 35

- Pupils represent the tens by stem and the ones by leaves.
- Pupils draw a vertical line with the tens values on the left of the line and the ones values on the right side.

i.e.

Stem	Leaf
1	2 4 6 6 8 9
2	0 1 3 5 5 5 6 7 9
3	1 4 5

Q4. The data below is the heights of 20 students measured to the nearest degree.

111	112	123	136	128	129	182	111	118	142
135	141	117	127	124	118	145	112	117	137
180	127	120	131	122	119	133	111	134	132.

a) Display the information using stem and leaf plot.

b) Using the stem and leaf plot, determine the;

i) Mode ii) Median iii) Mean

Ans: a) Guide the pupils to arrange the numbers in ascending order.

i.e. 111 111 111 112 112 117 117 118 118 119

120 122 123 124 127 127 128 129

131 132 133 134 135 137

141 142 145

180 182.

- Guide pupils to represent the tens by stem and leaf by ones.
- Pupils draw a vertical line with the tens values on the left of the line and the ones on the right side.

Stem	Leaf
11	1 11 2 2 7 7 8 8 9

12	0 2 3 4 7 7 8 9
13	1 2 3 4 5 6 7
14	1 2 5
18	0 2

b)i)From the plot, pupils observe that the most occurring number is 111.

Thus, the mode = 111

ii) From the plot, pupils observe the median as $\frac{127 + 127}{2} = \frac{254}{2}$ = 127 (i.e. there are two middle numbers occupying the 15th and 16th positions)

iii) – Pupils find the sum of all the numbers

- Divide by the total number of items in the distribution as shown below;

$$\text{Mean} = \frac{(111 \times 3) + (112 \times 2) + (117 \times 2) + (118 \times 2) + 119 + 120 + 122 + 123 + 124 + (127 \times 2) + 128 + 129 + 131 + 132 + 133 + 134 + 135 + 136 + 137 + 141 + 142 + 145 + 180 + 182}{30}$$

$$= \frac{3874}{30} = 129.13$$

Q5. Distinguish between ratio and rate giving examples in each case.

Ans: Ratio refers to comparing two or more things of the same unit or measure, dimension or kind. For example Esi has 9 oranges and Ama has three oranges, then we can say that Esi has 6 oranges more than Ama. i.e. 9 is 6 more than 3 because 9 – 3 = 6. We can then say that Esi's oranges is three times as Ama's. 9 is 3 times 3, because 9/3 = 3. We then write this ratio 9:3 which is read as "ratio of nine to three" on the other hand, rate is comparison of one measure with another of a different dimension E.g. A man travels at a speed of 20km per hour. Ekua bought a loaf of bread for 30Gp.

Q6. Explain what is meant by proportion.

Ans: Proportion involves two equivalent ratios or rates. It is a mathematical statement which sets one rate equal to another. Example: 20km in 1 hour = 40km in 2hours = 60km in 3 hours.

Q7. Explain how you would guide upper primary pupils six to express the lengths of two sticks with height 24cm and 16cm into its lowest term.

Ans: Steps:

1) The ratio of their lengths is 24 : 16.
2) Divide each stick into 2 equal parts, thus 24 ÷ 2 = 12 and 16 ÷ 2 = 8. Therefore the ratio of their lengths = 12 : 8.
3) Divide both sticks into two equal parts again thus 12 ÷ 2 = and 8 ÷ 2 = 4.
4) Divide both sticks again into two equal parts, thus 6 ÷ 2 = 3 and 4 ÷ 2 = 2
5) Thus the ratio of their lengths = 3 : 2, the pupils realize that the simplest form of 24 : 16 = 3 : 2.

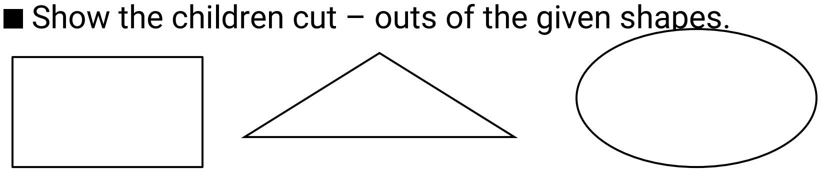
<u>Assignment.</u>

Guide pupils in the upper primary six to simplify the following ratios in their simplest form.

a) 6 : 9   b) 22cm : 33cm   c) 15 : 10cm.

Q8. Describe an activity in which you would engage kindergarten pupils to enable them identify the following shapes: rectangle, circle and triangle.

Ans: ■ Show the children cut – outs of the given shapes.



■ Help children to learn their names.

■ Give the cut – outs to the children to observe them carefully.

■ Let them pick from a collection one after the other a shape and name it.

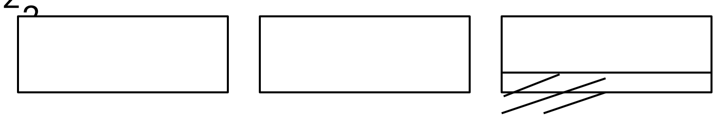
■ Let pupils identify a named shape from shapes drawn on a cardboard or a chalkboard.

■ Pupils call out the names of a shape and another pupils draws it or traces it from the cut outs.

■ Pupils identify or point to a a named shape in a real object.

Q9. Describe how you would use concrete material to show to pupils that $2\frac{1}{4} = \frac{9}{4}$

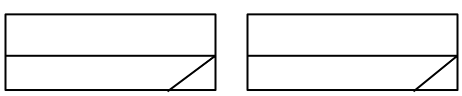
Ans: ▲take 2 identical sheets of paper and a fourth of other identical sheet to represent $2\frac{1}{2}$



i.e. $\frac{1}{4}$ (shaded portion)

▲Fold each of the two identical wholes into 4 equal parts and ask pupils to determine how many fourths there are in each.

i.e.



▲Pupils then determine the number of fourths there are altogether. i.e. 9 fourths.

▲Thus, pupils note that $2\frac{1}{4} = \frac{9}{4}$

Alternative solution

Pupils pick 2 purple rods of Cuisenaire rods another purple split into 4 whites and pick 1 white to represent $2\frac{1}{4}$.

Pupils split each of the purple rods into 4 whites to get 8 whites.

Pupil count the number of whites there are altogether.

Pupils get 9 whites.

Pupils compare 9 whites with a whole and obtain 9/4.

Thus they note that $2\frac{1}{4} = \frac{9}{4}$

ASSESMENT

Q1. What is diagnostic assessment? Explain briefly the purpose for using diagnostic assessment in teaching and learning mathematics in the lower primary school

Ans: A diagnostic assessment is one which is carried out to find out areas of strengths and weaknesses (difficulties) that child has. By this assessment the teacher gets to know of a pupils strengths and weaknesses and takes measures to encourage or rectify them or to select the appropriate method of teaching.

Q2. What is the difference between formative assessment and summative assessment?

Ans: Formative assessment is a continuous day to day assessment that take place through out the course of study. This is used as a feedback to correct errors and difficulties identified.

On the other hand, summative assessment is the type of assessment or evaluation that takes place at the end of a course or program which provides information about pupils level of achievement.

Q3. Explain the following terms i) Norm – referenced ii)Criterion – reference test.

a) Norm – referenced test is the type of assessment which compares pupil's performance with their peers. Examples, standardized test is a norm – reference test which compares pupil's performance with a large sample of pupils of similar age. Thus, a student's performance is assessed in relation to the group or norm.

b) Criterion – reference test measures the achievement of pupils against a given standard or criteria. Thus, pupils are assessed according to a given criteria. Example, a process test is a criterion reference test which usually has a fixed pass mark where pupils are expected to pass one test before carrying on to the next level, or the next class.

Q4. State two strengths and two challenges of criterion – referenced test.

Ans: STRENGTHS:

i) Test items have strong validity and reliability.

ii) Has well organized and reporting procedures.

iii) Assessment under CRT is comprehensive as it mostly covers all objectives outlined in the primary syllabus

iv) Test taken at the time through out the country.

WEAKNESS:

i) Resources available for it's administration is limited.

ii) Reports or feedbacks on the pupil's achievements are issued late

iii) Q5. Give three reasons why continuous assessment is useful.

iv) Ans: i) it provides a more representative sampling of pupil's performance both across time and across task. A student who is weak in one subject may perform better in other field work such as tailoring, cookery etc.

v) ii)Pupils are motivated to make continuous effort throughout the instruction period because all their grades contribute to the final grade they will obtain.

vi) iii)It makes an evaluation an integral part of the teaching process.

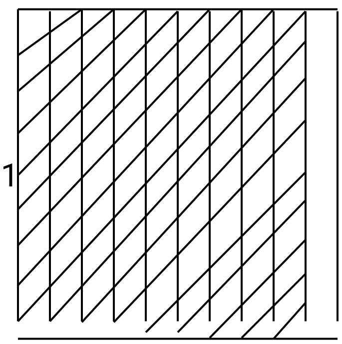
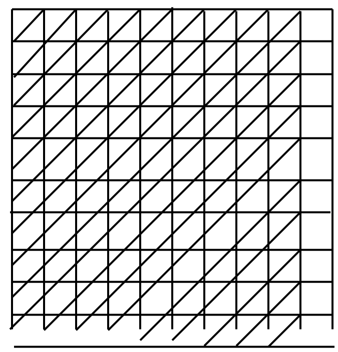
vii) iv)It gives a more comprehensive picture of the pupil's achievement than does a single final examination.

Q6. How would you explain to a pupil in primary 5 that 0.9 is greater than 0.61.

Ans: i) Explain that 0.9 is nine – tenths of ninety – hundredths, thus 0.9 $\frac{9}{10}$ $\frac{90}{100}$

And 0.61 is sixty – one hundredths is greater than sixty – hundredths

ii)Thus, pupils realize that ninety – hundredths is greater than sixty – hundredths.

fig 1  =  fig2.

$0.9 = \frac{9}{10}$ $0.9 = \frac{90}{100}$

iii) Pupils observe that the shaded portions in figure 2 is $\frac{90}{100}$ i.e. 90 small boxes.

v) Pupils compare the 90 small boxes to 61 = $\frac{61}{100}$ small boxes to realize that $\frac{90}{100} > \frac{61}{100}$ Thus they note that 0.9 > 0.61.

Assignment.

How would you explain to a pupil in primary 5 0.6 is greater than 0.39? (DBE Sept 2008).

LIKELY EXAMINATION QUESTIONS

1. Give and explain briefly TWO reason why children are taught mathematics in school.

2. Explain briefly TWO advantages of relational learning in mathematics.

3. Explain the multiple embodiment principle in the teaching and learning mathematics given example.

4. List Polya's FOUR steps in problem solving processes.

   b use a suitable example to explain each step.

5. What is the implication of Bruner's theory of development for teaching a new mathematics concept?

6. Give FOUR characteristics of the children in the pre – operational stage of piaget's stage of development and concrete operational stages of piaget's stages of development.

7. Using suitable example, clearly distinguish between a primary concept and a secondary concept in mathematics.

8. What does the mathematical process 'abstracting' mean?

9. a) What is a generic skill.

   b)Give TWO genetic skills and explain briefly how each is employed in teaching and learning of mathematics.

10. Explain briefly what is meant by each of the following in the teaching and learning of mathematics:

(a) Process objectives:

(b) Affective objective:

11. (a) Describe TWO different approaches you would use to help pupils in primary class 1 to solve the problem 6 – 2.

(b) Describe clearly how you would use a named base ten materials to help a primary class two pupil to add the two – digit numbers 56 and 47.

12. (a) What are pre – number activities?

(b) Give and explain two reasons why you would engage primary class 1 pupils in pre – number activities.

(c) Describe one activity for each of the following pre – number activities in which you would engage pupils:

(i) Sorting

(ii) Matching

(iii) Ordering

13. (a) Describe a game you would design for lower primary pupils for the consolidation of the concept of multiplication. Explain clearly how it is used.

(b) Describe clearly how you will use concrete materials to help pupils in primary class 3 to solve the division problem 12 ÷ 3 by (i) Sharing (ii) Grouping

14. (a) Describe an activity you would plan for primary class 6 pupils to enable them collect data in their classroom.

(b) Show and explain how you would guide the pupils to organize and represent the data in (a) in a bar graph, (b) block graph.

Q15. (a) Describe clearly an activity in which you would engage pupils in primary class 4 to enable them discover for themselves that the area Acm^2 of a rectangle with length 1 cm and width W cm is A = 1 × W.

16. Explain how you will guide an upper primary pupils to discover for herself that $3\frac{2}{5} = \frac{17}{5}$ using concrete materials.

(i) Write a story problem for $3 \div \frac{1}{4}$.

(ii) Show how you will guide your pupils to solve the problem $3 \div \frac{1}{4}$ using concrete materials.

17. Write down a story problem for a situation in which subtraction is described as (a)"missing addend." (b) take away. Explain how you would help a child solve the problem.

18. Describe briefly how you would explain to a class 2 pupil that in the number 132, the value of the digit 1 is greater than the digit 2.

19. What is conservation of number? Use suitable example to explain this term.

20. Using suitable examples, explain what is meant by an angle.

Describe clearly how you would guide the pupils to find pout which of the following is wider; the length of the classroom and the distance between two poles on the school compound.

21. Describe how you would use multi base block to solve 247 + 359.